1. Introduction

A demand for an ultra-precision stage in the field of nanotechnology has recently been increased, especially in AFM applications. Larger sized wafers require a stage to have fast response as well as long travel range maintaining high accuracy. Piezoelectric elements-driven ultra-precision stages have been used for high accuracy, fast response and high load capacity, which are allowable to apply the stages to the AFM applications.

Most of the piezoelectric elements-driven stages are guided by flexure hinges for force transmission and mechanical amplification\(^{1,2}\). However, the flexure hinge mechanisms cause lack of position accuracy due to coupled and parasitic motions. Therefore it is important that the mechanism design of the stage is focused on the stiffness of the flexure hinges to accomplish fast response and high accuracy without coupled and parasitic motions.

In this study, we present an optimal design of a piezoelectric elements-driven precision stage with two DOF. For the optimal design, a cost function is minimized by a Min-Max design algorithm\(^{3}\), subject to design constraints. Also, FEM analyses are carried out to certify the design requirements. Finally, experimental results show the validity of the designed mechanism.

2. Nano-positioning stage

The configuration of a nano-positioning stage driven by piezoelectric elements is shown in Fig. 1. The stage has the structure of a monolithic nested-loop type moving plates, in which each moving plate is guided by two four-link mechanisms. The four-link mechanism plays roles of motion guidance for translation as well as transmission of a preload to the piezoelectric element. The four hinges of the four-link mechanism are implemented by four round-notched flexure hinges. Due to the nested-loop type structure, the moving plates are actuated independently each other by piezoelectric elements so that the stage can avoid coupled interference motions.

![Fig. 1 Structure of two-axis nano-positioning stage](image)

3. Mathematical model

The flexure hinges in the four-link mechanism deform along both axial and rotational directions.
when the plate moves along translational direction. However the axial deformation of the flexure hinges are so small compared to the small translation of the plate that they may be ignored. Assuming that the flexure hinges operate as rotational springs, the inner and the outer plates are rigid bodies, and dimensions of the four four-link mechanisms are the same. The masses of the inner and the outer plates are $M_1$ and $M_2$. In the four-link mechanism, the distance between two rotational joints connecting the rotational link is $L$, and the mass and the inertia moment of the rotational link are $m$ and $I$. The equations of motions are decoupled as follows:

$$
\begin{align*}
(M_1 + M_2 + m + 4 \frac{L}{I}) \theta_x + 8 \frac{k_{th}}{L} x &= 0 \quad (1) \\
(M_2 + m + 4 \frac{L}{I}) \theta_y + 8 \frac{k_{th}}{L} y &= 0 \quad (2)
\end{align*}
$$

where $k_{th}$ is the rotational stiffness of the flexure hinge.

The rotational stiffness of the round-type flexure hinge, $k_{th}$ is given by

$$
k_{th} = \frac{2 Eb t^{3/2}}{9 \pi r^{1/2}}
$$

where $E$, $b$, $r$ and $t$ are Young’s modules, the thickness of the plate, the radius of the hole and the distance between two holes.

4. Design of nano-positioning stage

This stage is to measure the critical depth (CD) of nano patterns on a silicon wafer. For this task, the stage requires specifications as listed in Table 1. In Table 1, the working range and the first resonance frequency must be considered in design indices.

The angular errors consist of parasitic motion error, a machining error and an assembly error. The parasitic motion error can be investigated through simulation of the mechanism, however the machining error and the assembly error can be measured after machining and assembling the stage.

<table>
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<th>Table 1 Required specifications</th>
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<td>Working range</td>
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<td>Resolution</td>
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The resolution of the stage is decided by the resolution of position sensor, the input and output resolution of the amplifier for a piezoelectric element, a control algorithm and the resolution of D/A and A/D converters. Therefore the specifications both of the first resonance frequency and the working range cause following constraint conditions:

$$
\begin{align*}
f_1 &\geq 200 \text{ Hz} \quad (4) \\
K &\leq \frac{1}{10} k_{pol} \quad (5) \\
\sigma_{max} &\leq \frac{1}{10} \sigma_Y \quad (6)
\end{align*}
$$

where $f_1$, $K$, $k_{pol}$, $\sigma_{max}$ and $\sigma_Y$ are the first resonance frequency, the translational stiffness of the stage, the stiffness of the piezoelectric element, the maximum stress and the yield stress, respectively. To simplify the design problem, let the parameters $M_1$, $M_2$, $m$, $I$ and $b$ be fixed. Then, the hole radius $r$ and the distance $t$ between the holes are obtained by a design algorithm in following additional constraint conditions:

$$
\begin{align*}
r_1 &\leq r \leq r_2 \quad (7) \\
t_1 &\leq t \leq t_2 \quad (8)
\end{align*}
$$

In this paper, the optimal design parameters $r$ and $t$ were obtained by Min-Max design algorithm. Design indices $w_1$ to $w_4$ are chosen as follows:
where \( f_d \) and \( K_d \) are desired resonance frequency and desired stiffness. The design indices \( w_1, w_2 \) and \( w_3 \) are in favor of the minimum value whereas the design index \( w_4 \) is in favor of the maximum value in the constraint conditions. After the design indices are normalized, they are weighted. Then a composite design index \( W_c \) can be found as follows:

\[
W_c = \tilde{w}_1 \land \tilde{w}_2 \land \tilde{w}_3 \land \tilde{w}_4
\]

where \( \tilde{w}_i \) is \( i \)-th weighted normalized design index and the symbol \( \land \) is a fuzzy intersection. The optimal design parameters \( r \) and \( t \) are the maximal position of \( W_c \) in the constraint conditions. In this paper, we obtained the design parameters as \( (r, t)_{opt} = (4.9, 1.8) \text{ mm} \) (14)

5. Analyses of the designed stage

The resonance frequencies, the stiffness, the maximum stress calculated from the mathematical model using (14) are listed in Table 2. The calculated values satisfy the constraint conditions (4)~(6).

| \( f_1, f_2 \) | 201, 356 Hz |
| \( K \)       | 6.0 N/\( \mu \text{m} \) |
| \( \sigma_{\text{max}} \) | 49.8 MPa |

Also, the values in Table 2 were compared to the results of FEM analyses. Fig. 2 shows the results of dynamic analyses. Fig. 2-(a) and (b) show that the first and the second modes are in accordance with the calculated values. Fig. 2-(c) and (d) show that the frames of plates deform in high frequency, which violates an assumption that the plate is rigid.

The static analyses of the stage were also carried out. Fig. 3 shows the static deformation shapes applied by forces at x- and y-axis, and the weight of the stage. The stiffnesses based on the deformation and the applied forces are 5.97 N/\( \mu \text{m} \) and 5.96 N/\( \mu \text{m} \), in x- and y-axis, respectively. The stiffnesses of x- and y-axis accord the calculated results in Table 2.

The maximum stress is generated at the flexure hinge when the moving plates arrive at the maximum displacement. The analyzed maximum stress is 42.2...
6. Experiments

The developed stage is shown in Fig. 4. The first resonance frequency measured without the piezoelectric elements is 197 Hz, which is a little lower than the analyzed one. When the piezoelectric elements are attached at the stage, the first resonance frequency is increased to 212 Hz.

Fig. 4 Developed stage

Fig. 5 shows the stepwise response of x-axis. The steady-state response shows that the resolution of this stage is 5 nm.

Fig. 5 Stepwise response

The angular errors were measured as shown in Fig. 6. The angular errors are lower than the required specification.

Fig. 6 Measured angular errors

7. Conclusion remarks

The nano-positioning stage for CD measurement was developed. The stage was designed by Min-Max optimal design algorithm, and analyzed by calculations and FEM analyses. The analyzed results satisfied the design requirements. Also, experiments showed the validity of the designed stage.

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References