ALLOCATING TOLERANCES FOR VEE-GROOVE FIBER ALIGNMENT

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Abstract

This paper presents a method for allocating tolerances to dimensions in micro scale vee-grooves used to align optical fibers. The objective is to reduce the manufacturing cost without exceeding a limit on the misalignment between two mating fibers. The allocation procedure is performed on a 2D geometric model of a connector that aligns an array of multiple fibers. An analytical model of the connection, based upon statistics, is used for providing a relation between variation in manufactured dimensions and variation in the resulting misalignment of the fibers contained in the array. Optimal tolerances are determined using a non-linear constrained optimization algorithm that minimizes the manufacturing cost while satisfying constraints on the variation of the misalignment of any pair of fibers in the array. The method provides a useful tool when designing mass-produced connectors for multi-fiber cables, for which manufacturing cost and accuracy are critical parameters.

***Keywords*:** tolerance allocation, vee-grooves, optical fibers, alignment, manufacturing cost

Introduction

Vee-grooves are widely used in microscale devices, especially for positioning cylindrical objects. A common application is to align optical fibers, which is of primary interest for communications [1]. When connecting two fibers, a lateral misalignment, due to the manufacturing errors of the vee-groove, generates a consequential amount of signal loss. Considering Marcuse's [2] model for signal loss due to lateral misalignment of single-mode fibers, the cores of two fibers should be aligned within about 1 μm to achieve ~ 0.21 dB signal loss. This is a challenging proposition for only two individual fibers, but the challenge is even greater for mass produced interconnects which join cables containing eight or more fibers, as shown in Fig 1.

Our objective is to select tolerances that are sufficient for aligning optical fibers without excessive loss, but simultaneously minimizing manufacturing costs that arise from excessively tight tolerances. Tolerance allocation is generally formulated as an optimization problem with an objective function and set of constraints. In this case, the objective is to minimize the manufacturing cost which is a function of the tolerances. Tolerance relations for etched silicon vee-grooves are not available. However, for other materials like zirconium, a secondary material removal process like grinding may improve the tolerances. In this case, we employ relations developed by Chase [3] to relate manufacturing costs to tolerances. Both cost and signal loss can be defined as functions of the tolerances allocated by the designer. The optimization constraints are formulated as maximum tolerable signal loss within an entire connector.

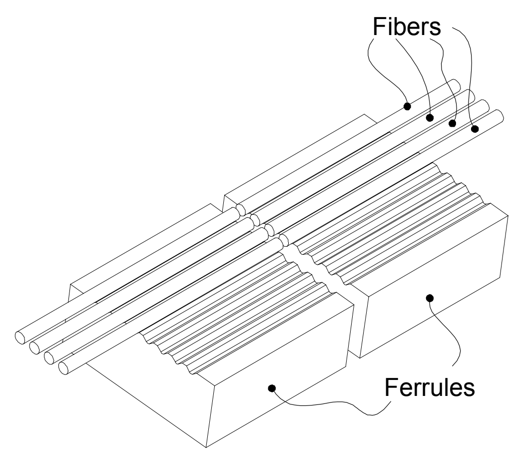

Fig 1: Multifiber Connector with Vee-Grooves

In formulating the optimization problem, the greatest challenge is determining a relationship

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between the tolerances and the performance criteria. For complex assemblies such as a fiber-optic connector, Monte Carlo simulations are effective means for relating final tolerance of an assembly to the tolerances of the components [4]. However, it may be difficult to implement Monte Carlo simulations within the optimization algorithm due to computational time. We instead use an alternative approach in which a few Monte Carlo simulations proved a mathematical model relating assembly tolerances to component tolerances. Knowing the geometry and the dimensions of the vee-groove, it is possible to define the signal loss of the fiber connection as a function of the tolerances of the vee-groove.

This paper presents a process to efficiently allocate the tolerances for vee-groove fiber alignment. The first step is to construct a mathematical model of the dimensions and geometry of vee-grooves. The second step is to define, through a statistical study, the misalignment of the fiber as a function of the tolerances in the vee-grooves. The third step is to estimate a relation between the tolerances and the manufacturing costs. Finally, tolerances are allocated with an optimization algorithm that minimizes the manufacturing cost for a given maximum limit on signal loss. An example illustrates the method.

Mathematical Model of the Dimensions and Geometry of a Vee-Groove

Fiber-to-fiber connections are a major source of optical loss. There exist three types of connection losses [1] that are directly related to the manufacturing errors within the connectors. The first one is caused by the lateral misalignment, due to the offset of the centerlines of the mating fibers. The second comes from the end-separation misalignment, which is the gap between the ends of the connected fibers. And finally, the third loss is generated by the angular misalignment, which occurs when there exists an angle between the two axes of the fibers.

The lateral misalignment is of most concern for connection loss since angular misalignment is negligible and end-separation is usually resolved by mechanical contact between the fibers or index-matching compounds. In considering only lateral misalignment, a 2D model of the vee-grooves is a reasonable approximation for representing the vee-groove geometry. The modeling plane is the *xz*-plane, *y* being the direction along the axes of the fibers. Geometry of a ferrule using vee-grooves is illustrated in the plane perpendicular to the axes of the fibers in Fig 2.

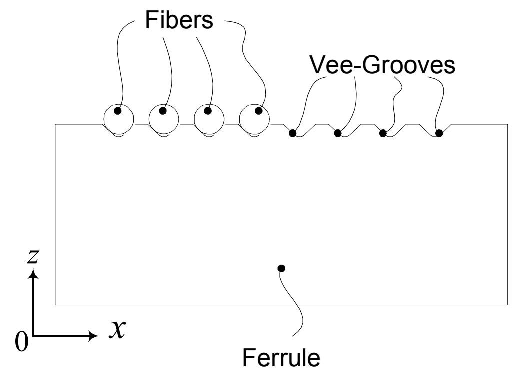

Fig 2: Connector in the *xz*-Plane

In the 2D configuration, it is possible to establish a mathematical relation between the variation of the dimensions in the ferrules and the lateral misalignment between two mating fibers.

As shown in Fig 3, the j[th] vee-groove of an array can be parametrically represented in two dimensions with four parameters and their manufacturing errors:

1. aperture angle, α_j, and angle error, δ_{α_j},
2. inclination angle, γ_j, which is the angle between the groove's bisector and a vertical line (ideally γ equals zero), and angle error, δ_{γ_j},
3. depth to the virtual vertex, h_j, and error, δ_{h_j}, and
4. radius of curvature, r_j, at the bottom of the groove and error, δ_{r_j}.

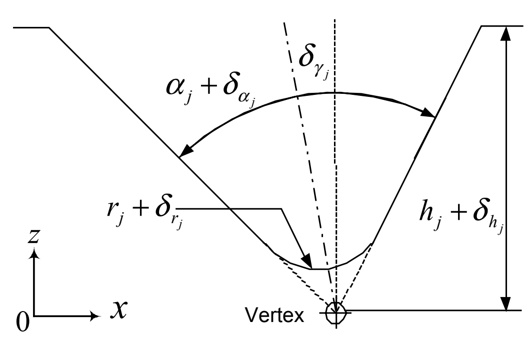

Fig 3: Manufacturing Errors in a Vee-Groove

Metrology applied to a ferrule cannot directly measure the value of the inclination angle, so it was decided to represent the aperture angle and the inclination angle as a combination of two half-angles: β_{Lj} on the left side and β_{Rj} one the right side. The aperture angle is defined as the sum of the two half-angles while the inclination angle is calculated as half their difference.

The geometry of a single vee-groove is then defined by four dimensions (β_{Lj}, β_{Rj}, h_j, r_j) and their variations. Furthermore, variation in the pitch between two successive vee-grooves is also a critical parameter for aligning fibers. The pitch p_j is then a fifth dimension used for modeling the connector.

Variation Analysis by the Law of Error Propagation

Tolerance allocation requires a relation between dimensional variation and connection loss. Marcuse [2] presented a relation, given in Eq (1), for the connection loss, T_j, of single-mode fibers as a function of the lateral misalignment, d_j, and the width parameters, w_{j1} and w_{j2}, of the two fibers. The dimensions, d_j, w_{j1}, and w_{j2} are expressed in the same length unit, and T_j has no dimension. Usually, connection loss is expressed in decibels.

$$T_j = \left(\frac{2w_{j1}w_{j2}}{w_{j1}^2 + w_{j2}^2} \right)^2 \exp\left[-\frac{2d_j^2}{w_{j1}^2 + w_{j2}^2} \right] \qquad (1)$$

For multi-mode fibers experimental data are used to establish the relation between lateral misalignment and connection loss, as shown in Fig 4.

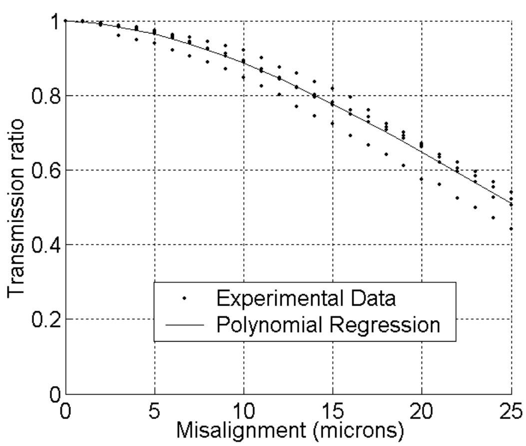

Fig 4: Experimental Determination of a Relation between Loss and Misalignment by Curve Fitting

For both single-mode and multi-mode cases, it is necessary to establish a relation between dimensional variation and lateral misalignment. This is done by applying the law of error propagation [5] on the mathematical model of the connectors. This method is computationally efficient when used in a tolerance allocation algorithm.

The geometric model of the vee-groove must be expressed in terms of statistics. Every dimension ξ_i is defined as a randomly distributed variable. Its mean μ_i equals the value of the nominal dimension, while its standard deviation σ_i is a third of the tolerance. For a complete representation of the connector, the same procedure is applied for the dimensions of the fibers and for the dimensions of the system that aligns the two ferrules. The distribution of errors between the two ferrules may be measured experimentally or predicted using another geometric/variation model.

The lateral misalignment d_j for the j[th] pair of fibers is modeled as a vector in the *xz*-plane. It is possible to define its coordinates, $\left(x_{d_j}, z_{d_j}\right)$, as a function of the dimensions of the vee-grooves, the fibers and the aligning system, as shown in Eqs (2)-(3).

$$x_{d_j} = f_{x_j}\left(\xi_1, \xi_2, \ldots, \xi_i, \ldots, \xi_n\right) \qquad (2)$$

$$z_{d_j} = f_{z_j}\left(\xi_1, \xi_2, \ldots, \xi_i, \ldots, \xi_n\right) \qquad (3)$$

n being the total number of assigned dimensions within the connector.

According to the law of error propagation, if the dimensions are independent (which is a reasonable assumption for most applications), then the standard deviation $\sigma_{x_{d_j}}$ of the lateral misalignment in the *x*-direction is given by Eq (4). A similar equation gives the lateral misalignment in the *z*-direction.

$$\sigma_{x_{d_j}}^2 \approx \sum_{i=1}^{n} \left(\frac{\partial f_{x_j}}{\partial \xi_i} \right)^2 \sigma_i^2 \qquad (4)$$

For perfect dimensions, the misalignment equals zero. Hence for random dimensions, its variation is directly related to its standard deviation. The law of propagation error then gives a direct analytical expression of the variance in lateral misalignment as function of the variances in the different dimensions of the connector.

Four dimensions, β_{Lj}, β_{Rj}, h_j and p_j, define the geometry of a vee-groove. Since the current study analyzes the sensitivity of the lateral misalignment to the geometry of the vee-grooves, the standard deviations of the components in the *x* and *z* directions for the lateral misalignment are expressed as shown in Eqs (5)-(6):

$$\sigma^2_{x_{d_j}} \approx \left(\frac{\partial f_{x_j}}{\partial \beta_{Lj}}\right)^2 \sigma^2_{\beta_{Lj}} + \left(\frac{\partial f_{x_j}}{\partial \beta_{Rj}}\right)^2 \sigma^2_{\beta_{Rj}} + \left(\frac{\partial f_{x_j}}{\partial h_j}\right)^2 \sigma^2_{h_j} + \left(\frac{\partial f_{x_j}}{\partial p_j}\right)^2 \sigma^2_{p_j} + Const_{x_j} \quad (5)$$

$$\sigma^2_{z_{d_j}} \approx \left(\frac{\partial f_{z_j}}{\partial \beta_{Lj}}\right)^2 \sigma^2_{\beta_{Lj}} + \left(\frac{\partial f_{z_j}}{\partial \beta_{Rj}}\right)^2 \sigma^2_{\beta_{Rj}} + \left(\frac{\partial f_{z_j}}{\partial h_j}\right)^2 \sigma^2_{h_j} + \left(\frac{\partial f_{z_j}}{\partial p_j}\right)^2 \sigma^2_{p_j} + Const_{z_j} \quad (6)$$

where the constant terms are due to the variations in the dimensions of the fibers and the aligning system.

Eqs (5)-(6) return the variances of the components in the *x* and *z* directions for the lateral misalignment, but Marcuse's model and Eq (1) require the magnitude of the misalignment, d_j. Its value could be expressed with a joint probability distribution for x_d and z_d, but an unknown correlation coefficient between the two components compromises the accuracy of the calculation. Therefore a Monte Carlo simulation of the connector is used to determine an empirical relation between the connection loss and the standard deviations of x_{d_j} and z_{d_j} by a two-step process.

The first step consists in collecting data from the Monte Carlo simulation. Its inputs are the nominal values and the tolerances of the different dimensions defining the geometry of a connector. A large number of connectors are virtually generated using the mathematical model previously presented. Their dimensions are normally distributed, with a mean equal to their nominal value and a standard deviation equal to one third of their tolerance. The algorithm calculates the misalignment of each randomly generated sample, then it performs a statistical treatment on the collected results. Finally, it returns the standard deviation of the components in *x* and *z* of the lateral misalignment, as well as a cumulative distribution function (cdf) of the connection loss (in dB) for every pair of fibers, as shown in Fig 5.

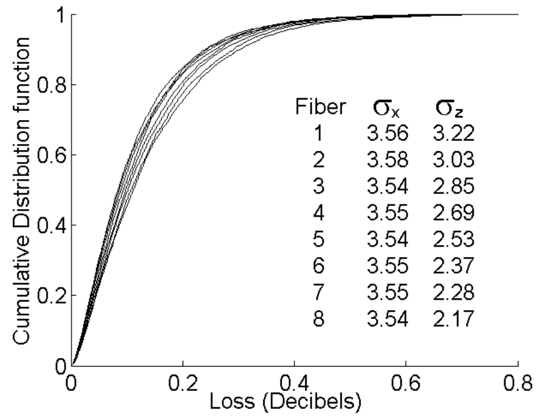

Fig 5: Outputs of Monte Carlo Simulation

Every cdf is curve fitted with a two-variable continuous function. Since the tolerance analysis focuses on the highest part of the cdf (beyond 90%), the curve fitting is performed exclusively on this part of the cdf, in order to get more reliable approximations. It has been found that for single-mode fibers, the cdf of a Weibull random variable is a good approximation, while a Gamma incomplete function fits well the cdf of the multi-mode fibers.

The simulation is run many times, with different input tolerances. The resulting cdf's are reduced to two parameters defining the fitted curve. Hence the first step of the process returns a set of values for the two fitting parameters, in function of the standard deviations of x_{d_j} and z_{d_j}.

The second step is a new curve fitting procedure. This time, one of the fitting parameters is plotted in function of the standard deviations of x_{d_j} and z_{d_j}, and it is curve fitted. The resulting relations are finally compared to new Monte Carlo simulations, and it has appeared that they were extremely reliable. These functions are used as empirical models of the connection loss.

Thereby a variation analysis based upon the law of propagation error, followed by an empirical yet accurate model of the connector performance, provides a relation between the connection loss and the tolerances of the vee-grooves.

Cost-Tolerance Relations

The cost of a manufactured part depends upon the selected manufacturing process and dimensional tolerances. The cost of achieving a particular tolerance depends upon both the dimension's nominal value and tolerance. The manufacturing cost generally increases if the tolerance is tightened, and it is more expensive to make a given tolerance on a large nominal dimension. Based on this, Chase [3] recommends expressing tolerances as reciprocal power functions for material removal processes. Eq (7) expresses the tolerance for the i^{th} dimension, t_i, as a function of cost, C_i, range, R_i, and three constants a_i, b_i, and c_i. The values of the three constants depend upon the range and the manufacturing process. Although a constant term would be necessary for accuracy, it is practically impossible to evaluate and doesn't affect the tolerance allocation.

$$t_i = c_i \times \frac{R_i^{a_i}}{C_i^{b_i}} \tag{7}$$

Similar functions are not available for etching processes commonly used with silicon.

Knowing the range and the manufacturing process of every dimension enables generating the cost-tolerance functions required to estimate the manufacturing cost of a connector in function of the tolerances assigned to its different dimensions. The portion of the total manufacturing cost that is attributable to vee-groove tolerancing is then the sum of the costs for the four dimensions, β_{Lj}, β_{Rj}, h_j and p_j, as shown in Eq (8).

$$Cost = \left(\frac{c_{\beta_{Lj}} \cdot R_{\beta_{Lj}}^{a_{\beta_{Lj}}}}{t_{\beta_{Lj}}}\right)^{1/b_{\beta_{Lj}}} + \left(\frac{c_{\beta_{Rj}} \cdot R_{\beta_{Rj}}^{a_{\beta_{Rj}}}}{t_{\beta_{Rj}}}\right)^{1/b_{\beta_{Rj}}} + \left(\frac{c_{h_j} \cdot R_{h_j}^{a_{h_j}}}{t_{h_j}}\right)^{1/b_{h_j}} + \left(\frac{c_{p_j} \cdot R_{p_j}^{a_{p_j}}}{t_{p_j}}\right)^{1/b_{p_j}} \tag{8}$$

When tolerancing the connector, only one tolerance is assigned to a nominal dimension, even if this feature is repeated several times in the product. That is the reason why the manufacturing cost depends upon the dimensions of one single vee-groove, instead of the complete array of grooves.

Tolerance Allocation by Optimization

Optimal tolerances for the dimensions are determined using nonlinear constrained optimization. The problem is formulated as a minimization subject to constraints. The function to minimize is the manufacturing cost of the connector with respect to its tolerances, as defined in the previous section. Constraints are formulated by specifying that the standard deviation of the lateral misalignment, σ_{d_j}, for every pair of fibers within the connector must be positive yet below a critical value. Additional bounds can be specified to prevent the optimization from driving the assigned tolerances to unreasonably high or low values.

Since this optimization only deals with allocating tolerances to the vee-grooves, it is assumed that the tolerances for the fibers and the system that aligns the ferrules is already known empirically or predicted by another analysis. The variables of the optimization problem are then the tolerances for the four dimensions (β_{Lj}, β_{Rj}, h_j, p_j) defining the vee-grooves. The radius of curvature is not included, since it doesn't affect the positioning of the fiber.

This method was used to allocate tolerances to an exemplary connector. The objective was to minimize the manufacturing cost of an 8-fiber connector while the connection loss of every pair of single-mode fibers should be less than 0.5 dB. The calculated connection losses along the connectors are displayed in Fig 6, and the resulting tolerances allocated by the optimization procedure are listed in Table 1.

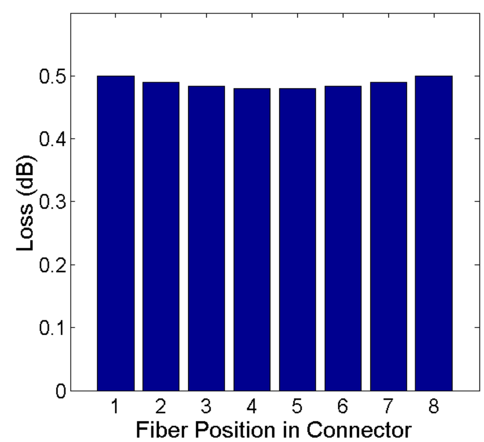

Fig 6: Computed Losses

Table 1: Computed Tolerances

Dimension	Assigned Tolerance
Left Angle	5.76×10^{-3} radians
Right Angle	5.76×10^{-3} radians
Depth of the Vertex	0.519 microns
Pitch between Vees	0.594 microns

Conclusions

Manufacturing cost is a critical parameter for mass-produced features. On the other hand, accurate devices need low variation in their dimensions. Mass-produced multi-fiber connectors using vee-grooves face both problems. Their design can be optimized by a tolerance allocation method.

This paper presents a method for allocating tolerances to the dimensions of the vee-grooves. A mathematical model of the geometry is constructed and used in a statistical analysis. Applying the law of error propagation allows derivation of a relation between dimensional variance and variance in signal loss. Optimal tolerances of the vee-grooves are computed by minimizing the relative manufacturing cost while respecting constraints on maximum loss acceptable for every pair of mating fibers. The method is demonstrated for an exemplary 8-fiber vee-groove alignment system.

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