Phase-measuring volumetric interferometer for three-dimensional coordinate metrology

Seung-Woo Kim*, Hyug-Gyo Rhee, Ji-Young Chu
Department of Mechanical Engineering
Korea Advanced Institute of Science and Technology
Science Town, Daejeon, 305-701, KOREA
*: Who is responsible for correspondence.
Tel: ++82-42-869-3217; Fax: ++82-42-869-3210; E-mail: swk@kaist.ac.kr

1. Introduction

In designing precision machines, it is required that displacement transducers be installed with as small Abbe offsets as possible [1]. However, the requirement is usually not satisfactorily met with widely used optical encoders and laser interferometers especially for three-dimensional coordinate measuring machines. To address the problem, the authors have recently proposed the new concept of phase-measuring volumetric interferometer that enables to accurately measure the xyz-coordinates of the probe without metrology frames [2,3]. The interferometer is composed of a movable target and a fixed photo-detector array. The target is made of point diffraction sources to emit two spherical wavefronts, whose interference is monitored by an array of photo-detectors. Phase shifting is applied to obtain the precise phase values of the photo-detectors [4,5]. Then the measured phases are fitted to a geometric model of multilateration so as to determine the xyz-location of the target by minimizing least square errors [6,7]. For description of the interferometer’s geometric configuration, the global xyz-coordinate system is set so that its origin is positioned right on one of the photo-detectors as illustrated in Figure 1. The two point diffraction sources on the target are given the coordinates as \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\), respectively, which are to be determined to find out the xyz-location of the target. The proposed interferometer has been designed and built with a volumetric uncertainty of less than 1.0 \(\mu m\) within a cubic working volume of side 100 mm. Here, in this paper we also present error sources, an evaluated uncertainty, and test results from the prototype system. Self-calibration concept of two-dimensional precision metrology stages [8,9] is applied to prove the performance of the interferometer.

2. Phase-measuring volumetric interferometry

The interferometric intensity field from two spherical wavefronts, referred to as \(u_1\) and \(u_2\), respectively, is derived as

\[
I = \left| u_1 + u_2 \right|^2 = \Pi + \Gamma \cos(\Phi + \Delta \phi)
\]

where \(\Pi = \frac{U_1^2}{r_1^2} + \frac{U_2^2}{r_2^2}\), \(\Gamma = \frac{2U_1 U_2}{r_1 r_2}\), \(\Phi = \frac{2\pi}{\lambda} (r_1 - r_2)\), and \(\Delta \phi \equiv (\phi_1 - \phi_2)\)

Figure 1. Geometry of the proposed volumetric interferometer.
Eq. (2) indicates that the six unknowns \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) can be determined by solving inverse kinematics if the absolute value of \(\Phi\) is provided from more than six different locations. For that, a two-dimensional array of photo-detectors is deployed to capture the interferometric intensity \(I\) at multiple locations. From the measured intensity, the phase \(\Phi + \Delta \phi\) of Eq. (1) is computed by applying the well-established phase measuring technique with phase shifting. For description, let us introduce the superscript \(k\) so that \(\Phi^k\) refers to the computed value of \(\Phi\) at the location of \((x^k, y^k, z^k)\). Now, all the measured values of \(\Phi^k\) are processed to be unwrapped, starting from a particular reference principal phase value that is for convenience designated as \(\Phi^0\) at location \((x^0, y^0, z^0)\). Then, a new variable \(\Lambda^k\) is defined such as

\[
\Lambda^k = \frac{\lambda}{2\pi} \left[ (\Phi^k + \Delta \phi) - (\Phi^0 + \Delta \phi) \right] = \frac{\lambda}{2\pi} (\Phi^k - \Phi^0) \tag{3}
\]

Now, as the final step for absolute distance measurements, using the relationship of Eq. (3), the six unknowns \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) are determined so as to minimize the cost function that is defined as

\[
E = \sum_k \left[ \frac{\lambda}{2\pi} (\Phi^k - \Phi^0) - \Lambda^k \right]^2 \tag{4}
\]

Note that the summation over \(k\) in the above expression is performed all over the photo-detectors to be considered, and \(\Lambda^k\) represents the actually measured value of \(\Lambda^k\). The cost function \(E\) is so highly nonlinear in terms of the unknowns \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) that no explicit solutions exist for the minimization. Thus, numerical technique is used to search for the global minimum of the cost function[2],[3].

### 3. Error analysis

For the estimation of measurement uncertainty, a thorough analysis has been made following the ISO’s guide to the expression of measurement uncertainty [10]. The major sources of errors considered are listed in Table 1 along with their individual contributions determined through appropriate error models or experiments. All the error sources in Table 1 are divided into two groups according to whether their individual influences are directed to phase or displacement. All the listed uncertainties represent the worst cases that could possibly occur over the whole working volume and they are given in terms of standard deviation. The total uncertainty is worked out to be 420 nm for \(x\)-, 550 nm for \(y\)-, and 9.7 nm for \(z\)-directions when we use a \(640 \times 480\) CCD as a photo-detector array. The CCD array used for the evaluation has an overall dimension of 6.4 mm in the \(x\)-direction and 4.8 mm in the \(y\)-direction, so the larger \(x\)-dimension takes more phase information resulting in better uncertainty in the \(x\)-direction. The \(z\)-direction has the best result as the cost function defined in Eq.(4) turns out to be most sensitive to the \(z\)-coordinate.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Error sources</th>
<th>Input condition</th>
<th>Output uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase errors</td>
<td>Laser intensity stability</td>
<td>(\leq 0.07%) of (\Pi)</td>
<td>0.0071 rad.</td>
</tr>
<tr>
<td></td>
<td>Temperature variation in the fiber</td>
<td>(\leq 0.01)K</td>
<td>0.0084 rad.</td>
</tr>
<tr>
<td></td>
<td>Low frequency pressure variation in the fiber</td>
<td>(\leq 8 \times 10^{-4}) rad.</td>
<td>5 \times 10^{-5} rad.</td>
</tr>
<tr>
<td></td>
<td>Nonlinearity of fiber elongation &amp; PZT</td>
<td>(\leq 0.15) rad.</td>
<td>0.009 rad.</td>
</tr>
<tr>
<td></td>
<td>Electrical noise</td>
<td>(\leq 0.7%) of (\Pi)</td>
<td>0.005 rad.</td>
</tr>
<tr>
<td></td>
<td>Detector nonlinearity</td>
<td>(\leq 0.8%) of (\Pi)</td>
<td>0.023 rad.</td>
</tr>
<tr>
<td>Displacement errors</td>
<td>Sphericity of the spherical wave</td>
<td>(\leq 1 \times 10^{-5}) rad.</td>
<td>4 \times 10^{-3} nm</td>
</tr>
<tr>
<td></td>
<td>Laser frequency stability</td>
<td>(\leq 1) MHz</td>
<td>1.3 \times 10^{-6} nm</td>
</tr>
<tr>
<td></td>
<td>Air refractive index variation</td>
<td>(\leq 10^{-7})</td>
<td>6.3 \times 10^{-5} nm</td>
</tr>
<tr>
<td></td>
<td>Detector position error</td>
<td>(\leq 100) nm</td>
<td>8 \times 10^{-2} nm</td>
</tr>
</tbody>
</table>

This uncertainty analysis also reveals an important fact that from the viewpoint of the system design, the size of the CCD array is one of the most influencing parameters. The uncertainty especially in the \(x\)- and \(y\)-directions drastically improves as the size of the CCD arrays is enlarged. For instance, if a \(2048 \times 2048\)
array is used, the uncertainty in the x- and y-directions reaches 20 nm, almost the same level of the z-direction.

4. Measurement results

A prototype volumetric interferometer has been set up on a two-dimensional precision stage as illustrated in Figure 2. For testing, the target emitting two spherical wavefronts is fixed on the granite table and the CCD photo-detector array is attached to the moving stage. An artifact made of glass plate We also use the $7 \times 7$ array made by an electron beam lithograph system as. The readings of the optical linear scales installed on the measurement axes of the stage

Figure 2. Photographic view of the two-dimensional stage with the volumetric interferometer.

Figure 3. Deviation map between the optical scale reading and the volumetric interferometer result.

Figure 4. The reconstructed systematic error using the Fourier self-calibration algorithm.

(a) The systematic error of the two-dimensional optical scale.

(b) The systematic error of the volumetric interferometer.

array is attached to the moving stage. An artifact made of glass plate We also use the $7 \times 7$ array made by an electron beam lithograph system as. The readings of the optical linear scales installed on the measurement axes of the stage
are taken as the initial values of the numerical search of the interferometer, so the convergence to true \(xyz\)-coordinates can be accelerated. Figure 3 shows the differences between the readings of the optical scale and the results of the interferometer at each node point of the artifact plate. Test results were obtained within \(60 \text{mm} \times 60 \text{mm}\) working area, in which the maximum deviations were measured 0.48 \(\mu \text{m}\) for \(x\)- and 1.3 \(\mu \text{m}\) for \(z\)-direction. However, this deviation map does not mean the error of the volumetric interferometer because it includes many error sources such as the alignment error between two metrology frame, the optical scale’s systematic error, scale factor, and random measurement noise.

To remove these effects and extract the systematic error of the proposed interferometer, we applied the Fourier self-calibration algorithm\[9\]. When we obtained the coordinates of each node for calibration, we used averaging technique to reduce the random noise effect. Figure 4 (a) shows the systematic error of the two-dimensional optical scale, in which the absolute values of the maximum error were measured 2.0 \(\mu \text{m}\) for \(x\)- and 1.5 \(\mu \text{m}\) for \(z\)-direction. On the other hand, Figure 4 (b) shows the systematic error of the interferometer, in which the absolute values of the maximum error were measured 0.87 \(\mu \text{m}\) for \(x\)- and 0.82 \(\mu \text{m}\) for \(z\)-direction. Through a repeat test of self-calibration, we confirm that the reconstructed systematic error has reasonable repeatability.

5. Summary
A volumetric interferometer has been developed and tested. The interferometer consists of two optical fibers held on the target to emit two identical monochromatic spherical wavefronts with a lateral offset and a two-dimensional array of photo-detectors to monitor the interference of the two spherical wavefronts. The \(xyz\)-coordinates of each fiber’s end are determined by fitting the measured phase values to a geometric model of multilateration so as to minimize least square errors. A prototype demonstrates that the proposed interferometer is capable of measuring the \(xyz\)-coordinates of the target with a volumetric uncertainty of less than 1 \(\mu \text{m}\), which will be further improved to a level of 0.1 \(\mu \text{m}\) if large size photo-detector arrays are used. In addition, the systematic errors of the interferometer have been identified adopting the concept of self-calibration, and the maximum error was 870 nm within a working area of 60 mm x 60 mm x 20 mm.

6. Acknowledgment
The work published here has been conducted as part of the national Creative Research Initiatives program sponsored by the Ministry of Science and Technology of the Republic of Korea.

References