

THE VIRTUAL-SHAPE APPROACH FOR MODELING VOLUMETRIC ERRORS OF MACHINE TOOLS

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Abstract: A modeling method of volumetric errors of machine tools through the concept of a virtual-shape element is proposed. The virtual-shape element is introduced to exclude problems occurred owing to Abbe offsets. Hypothetical elements without having real dimensions are used for the virtual-shape elements. An equivalent diagram for error propagation through the virtual-shape element is studied as well. The proposed modeling approach can include both geometric and thermal errors. The derived error model requires no dimensional information of machine tools due to elimination of the Abbe error terms that are unavoidable in the previous method. Introducing the virtual-shape element can also solve the difficulties in the parameter identification process.

Keywords: Abbe error, Error synthesis, Equivalent vector approach, Geometric error, Machine tools, Separated element approach, Thermal error, Virtual-shape approach, Volumetric error

1. Introduction

Machining accuracy depends upon deviation from the planned relative movement between the tool and the workpiece. For three-axis machine tools and coordinate measuring machines, this relative error is called the volumetric error. Among the machine error sources, geometric and thermal errors are known to be key contributors [1,2]. In general, the error synthesis method using the homogeneous coordinate transformation is widely used to derive a volumetric error model [3].

The error synthesis method can be classified into two approaches with or without thermal errors: the equivalent vector approach and the separated element approach. The equivalent vector approach represents the configuration of machine tools or coordinate measuring machines with three equivalent vectors [4-6]. Each vector is modeled as a joint transformation matrix. Therefore, thermal errors of shape elements such as spindle errors cannot be properly incorporated into the model. The equivalent vector approach has been mainly used for modeling geometric errors. On the other hand, the separated element approach analyses the configuration of machine tools as a sequence of shape elements connected by joint elements [7-9]. Each shape and joint is modeled as a shape and joint transformation matrix, respectively. Therefore, both geometric and thermal errors can be modeled explicitly.

Although the separated element approach is able to

incorporate into thermal errors, volumetric error models derived by using this approach have several weak points: (1) The measuring line of laser interferometers for identification of model parameters has to be aligned on the same axes of ball screws because the setup positions of measuring instruments are restricted to the positions of local coordinate frames defined at the model building stage. However, it is difficult or impossible to fulfill the above-mentioned requirement practically. (2) The volumetric error model is more complex than that derived by using the equivalent vector approach because of the Abbe error terms expressed as multiplication of the rotational errors of joint elements by the real dimensions of shape elements. The reason why the Abbe error terms are intervened is that joint transformation matrices are defined as the errors along ball screws. (3) The volumetric error model requires dimensional information of machine tools for error compensation. It is not easy for users to obtain exact dimensional information of machine tools. All the above disadvantages are resulted from the fact that shape elements have actual size.

In this paper, a new concept of a virtual-shape element is introduced to compensate for disadvantages concerning both approaches. An equivalent diagram for error propagation through the virtual-shape element is studied as well. A volumetric error model for the C-shaped vertical machining center is derived by using the proposed method and compared with that derived

by using the previous methods.

2. The virtual-shape approach

As shown in Fig. 1, a shape element has translational and rotational error terms due to thermal errors. A shape transformation matrix that transforms the coordinate of a point in the $X_2'Y_2'Z_2'$ coordinate frame into the $X_1Y_1Z_1$ reference frame can be represented as follows:

$$S_{actual} = \begin{bmatrix} 1 & -\alpha(t) & \beta(t) & a + \Delta a(t) \\ \alpha(t) & 1 & -\gamma(t) & b + \Delta b(t) \\ -\beta(t) & \gamma(t) & 1 & c + \Delta c(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where, $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are rotational errors around Z-, Y- and X-axis, respectively. $\Delta a(t)$, $\Delta b(t)$ and $\Delta c(t)$ are translational errors in X-, Y- and Z-axis directions. a , b and c are dimensions of the ideal shape element in X-, Y- and Z-axis direction. All of the error terms are usually modeled as a function of time or temperature, t .

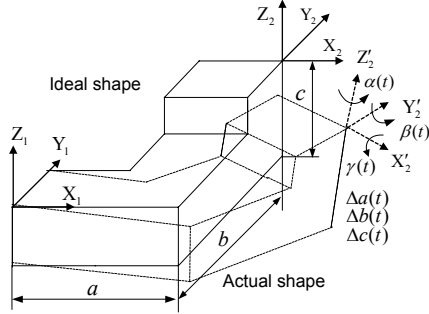


Fig. 1 Relative position of two coordinate systems established on a real shape element.

A virtual-shape element is defined as the hypothetical element that has no real dimensions. But, it is assumed that the hypothetical element can have shape errors such as translational and rotational errors as follows:

$$V_{actual} = \begin{bmatrix} 1 & -\alpha(t) & \beta(t) & \Delta a(t) \\ \alpha(t) & 1 & -\gamma(t) & \Delta b(t) \\ -\beta(t) & \gamma(t) & 1 & \Delta c(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

If there is no error, the virtual-shape transformation matrix could become an identity matrix. Note that a prerequisite for Eq. (2) is coincident with the position of $X_2Y_2Z_2$ coordinate frame as that of $X_1Y_1Z_1$ coordinate frame.

Fig. 2 shows an equivalent diagram for modeling

volumetric errors of a C-shaped vertical machining center by using virtual-shape elements. The reference coordinate system is established at the position where the origin of the workspace is joined with that of the spindle. The kinematic chain from the reference coordinate system to the tool is as follows: A virtual-shape element for Z-axis, V_z , exists between the origin of reference coordinate system, O_R , and the joint element for Z-axis, J_z . A virtual-shape element for a spindle, V_s , exists between the joint element for Z-axis, J_z , and the spindle origin, O_s . The kinematic chain from the reference coordinate system to the workpiece is determined in a similar way.

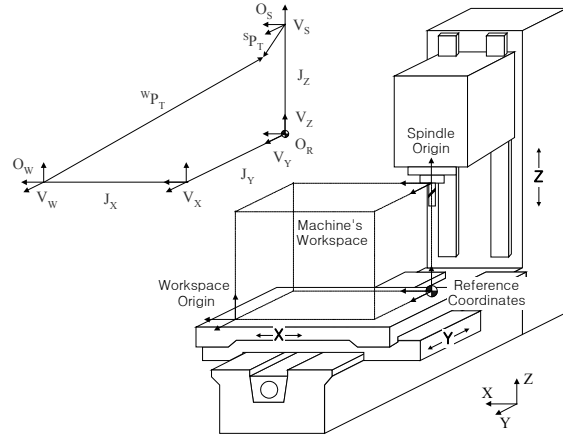


Fig. 2 Equivalent diagram in the virtual-shape approach.

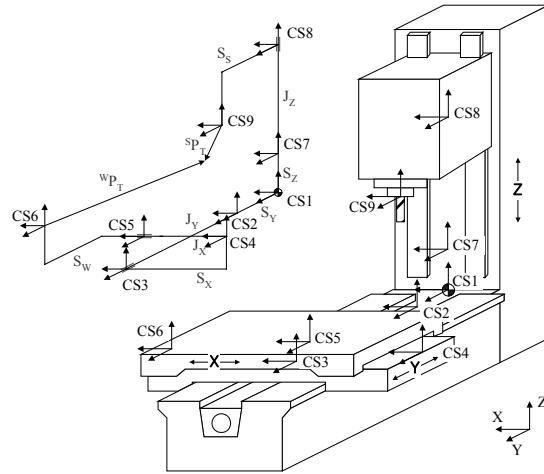


Fig. 3 Equivalent diagram in the separated element approach

Positions and roles of virtual-shape elements can be explained more easily by comparing two equivalent diagrams shown in Fig. 2 and Fig. 3. Fig. 3 shows an equivalent diagram for modeling volumetric errors of the same machining center by using the separated

element approach. The reference coordinate system, CS1, is established at the intersection point between Y- and Z-axis ball screws. A local coordinate frame CS7 is located at the lower mount of the Z-axis ball screw. CS8 is established at the nut of the Z-axis ball screw, CS9 is at the tool grasp point, and so on. So, a joint transformation matrix is defined with translational and rotational error terms along a ball screw axis in this separated element approach.

The equivalent diagram shown in Fig. 2 is acquired by moving the coordinate frame CS1 in Fig. 3 to the position where O_R in Fig. 2 is established, and substituting the virtual-shape elements for the real shape elements. Since the virtual-shape element has no real dimensions, CS7, CS9, CS2, CS4 and CS6 coordinate frames are transferred to CS1, CS8, CS1, CS3 and CS5 coordinate frames, respectively. And, since the reference coordinate frame is transferred from CS1 to O_R , the joint transformation matrix shown in Fig. 3 is redefined at the equivalent diagram shown in Fig. 2 with translational and rotational error terms along an axis of the reference coordinate frame O_R .

The tool position with respect to the origin of the workspace, ${}^w P_T$, can be derived from the equivalent diagram shown in Fig. 2 as follows:

$$\begin{aligned} {}^w P_T \text{ actual} &= [X+dX \ Y+dY \ Z+dZ \ 1]^T \\ &= [V_W]_{\text{actual}}^{-1} [J_X]_{\text{actual}}^{-1} [V_X]_{\text{actual}}^{-1} [J_Y]_{\text{actual}}^{-1} [V_Y]_{\text{actual}}^{-1} \\ &\quad \times [V_Z]_{\text{actual}} [J_Z]_{\text{actual}} [V_S]_{\text{actual}} {}^S P_T \end{aligned} \quad (3)$$

where, ${}^S P_T = [t_x \ t_y \ t_z \ 1]^T$ is a tool offset vector. dX , dY and dZ are volumetric error terms in X-, Y- and Z-axis directions, respectively.

3. Case study

In a vertical machining center, the dominant thermal error sources are the thermal drift and tilt of a spindle, the thermal drift and expansion of three ball screws, and the thermal distortion of C-shaped machine frames. Joint errors are divided into stationary geometric errors and time-variant thermal errors.

Considering the prescribed dominant error sources, the joint and virtual-shape transformation matrices can be expressed as follows:

$$[J_X]_{\text{actual}} = \begin{bmatrix} 1 & -\varepsilon_z(x) & \varepsilon_y(x) & x + \delta_x(x) + P_x(t)x \\ \varepsilon_z(x) & 1 & -\varepsilon_x(x) & \delta_y(x) + S_{xy}x + \Delta S_{xy}(t)x \\ -\varepsilon_y(x) & \varepsilon_x(x) & 1 & \delta_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$[J_Y]_{\text{actual}} = \begin{bmatrix} 1 & -\varepsilon_z(y) & \varepsilon_x(y) & \delta_x(y) \\ \varepsilon_z(y) & 1 & -\varepsilon_y(y) & y + \delta_y(y) + P_y(t)y \\ -\varepsilon_x(y) & \varepsilon_y(y) & 1 & \delta_z(y) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$[J_Z]_{\text{actual}} = \begin{bmatrix} 1 & -\varepsilon_z(z) & \varepsilon_x(z) & \delta_x(z) + S_{zx}z + \Delta S_{zx}(t)z \\ \varepsilon_z(z) & 1 & -\varepsilon_x(z) & \delta_y(z) - S_{yz}z - \Delta S_{yz}(t)z \\ -\varepsilon_y(z) & \varepsilon_x(z) & 1 & z + \delta_z(z) + P_z(t)z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$[V_X]_{\text{actual}} = \begin{bmatrix} 1 & 0 & 0 & \Delta a_x(t) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [V_Y]_{\text{actual}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \Delta b_y(t) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$[V_Z]_{\text{actual}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta c_z(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [V_W]_{\text{actual}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$[V_S]_{\text{actual}} = \begin{bmatrix} 1 & 0 & \beta_s(t) & \Delta a_s(t) \\ 0 & 1 & -\gamma_s(t) & \Delta b_s(t) \\ -\beta_s(t) & \gamma_s(t) & 1 & \Delta c_s(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

where, $\delta_i(j)$ and $\varepsilon_i(j)$ are translational and rotational error terms along an axis of the reference coordinate system, i represents error direction and j represents moving direction. x , y and z are travel distances. S_{ij} is a squareness error between i - and j -axis of the reference coordinate system. $P_x(t)$, $P_y(t)$ and $P_z(t)$ represent linear thermal expansions of X-, Y- and Z-axis, respectively. $\Delta a_s(t)$, $\Delta b_s(t)$ and $\Delta c_s(t)$ are thermal drifts of a spindle in X-, Y- and Z-axis directions, respectively. $\beta_s(t)$ and $\gamma_s(t)$ are tilt motions of a spindle. $\Delta a_x(t)$, $\Delta b_y(t)$ and $\Delta c_z(t)$ represent thermal drifts of each axis origin. $\Delta S_{xy}(t)$, $\Delta S_{yz}(t)$ and $\Delta S_{zx}(t)$ are thermal distortions of C-shaped machine frames.

Substituting Eqs. (4)~(11) into Eq. (3), and neglecting higher order terms, volumetric errors are derived as follows:

$$\begin{aligned} dX &= -\delta_x(x) - \delta_x(y) + \delta_x(z) - P_x(t)x + \Delta a_s(t) - \Delta a_x(t) \\ &\quad + \{\varepsilon_z(x) + \varepsilon_z(y) - \varepsilon_z(z)\}t_y - \{\varepsilon_y(x) + \varepsilon_y(y) - \varepsilon_y(z) - \beta_s(t)\}t_z \\ &\quad - \{\varepsilon_z(x) + \varepsilon_z(y)\}y - \{\varepsilon_y(x) + \varepsilon_y(y) - S_{zx} - \Delta S_{zx}(t)\}z \\ dY &= -\delta_y(x) - \delta_y(y) + \delta_y(z) - P_y(t)y + \Delta b_s(t) - \Delta b_y(t) \\ &\quad - \{\varepsilon_z(x) + \varepsilon_z(y) - \varepsilon_z(z)\}t_x + \{\varepsilon_x(x) + \varepsilon_x(y) - \varepsilon_x(z) - \gamma_s(t)\}t_z \\ &\quad + \{\varepsilon_z(x) - S_{xy} - \Delta S_{xy}(t)\}x + \{\varepsilon_x(x) + \varepsilon_x(y) - S_{yz} - \Delta S_{yz}(t)\}z \\ dZ &= -\delta_z(x) - \delta_z(y) + \delta_z(z) + P_z(t)z + \Delta c_s(t) + \Delta c_z(t) \\ &\quad + \{\varepsilon_y(x) + \varepsilon_y(y) - \varepsilon_y(z) - \beta_s(t)\}t_x - \{\varepsilon_x(x) + \varepsilon_x(y) - \varepsilon_x(z) - \gamma_s(t)\}t_y \\ &\quad - \{\varepsilon_y(x)\}x + \{\varepsilon_x(x) + \varepsilon_x(y)\}y \end{aligned} \quad (12)$$

4. Comparison with traditional approaches

Measuring devices such as a laser interferometer or other mechanical methods can identify the well-known 21 geometric errors. Fig. 4 shows two setup locations for identification of model parameters. The choice of

setup location for the laser and optics is usually determined according to what information is desired [10]. If the translational error along the tool path is the desired information, the optics should be positioned to the axis of tool path. However, if the translational error along the axis of ball screw is the desired information, the optical components should be positioned to the axis of ball screw. It should be remembered that these two setups are not equivalent and, in general, will not yield the same error results owing to Abbe offset. In the separated element approach, since the joint transformation matrix is defined along the axis of ball screw, the optics should be positioned to the axis of ball screw. However, it is difficult or impossible to fulfill this requirement practically. Lo et al. [11] have been proposed the flexible error synthesis model to acquire correct error values from measurements taken at different spatial positions.

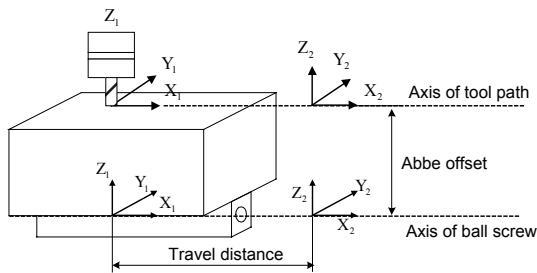


Fig. 4 Setup configuration of the laser and optics.

On the contrary, by introducing the virtual-shape element, a joint transformation matrix is redefined with errors along an axis of the reference coordinate frame established at the vertex of the workspace. Therefore, the error terms of each joint transformation matrix can be identified accurately through positioning the optical components at the origin of the reference coordinate system and measuring the errors during the movement along the axis of the reference coordinate system corresponding to the edge of the workspace.

A laser interferometer or dial gauges can also measure thermal drifts and expansions of three ball screws [12]. Measurement of thermal drift and tilt motions of a spindle is usually performed with a test mandrel and the fixture in which five displacement transducers are mounted [12]. The fixture should be set up at the origin of reference coordinate system even though the test is carried out on machines with rotating spindles. Heat generated by spindle rotation is transferred to the surrounding structure and mainly causes a variation of the column temperature distribution. Time-variant thermal distortions of machine frames affect the volumetric error as shown in Eq. (12). Therefore, in order to measure accurate drift data, the fixture should be securely fixed at the origin

of the reference coordinate system.

In the equivalent vector approach, since the reference coordinate frame is established at the same position with the virtual-shape approach, the choice of setup location for the laser and optics is identical with the above-mentioned measuring method. However, the equivalent vector approach can't be applied to model thermal errors of machine tools, because the equivalent vector approach represents the configuration of three-axis machine tool as three joint transformation matrices. That is, shape elements such as a spindle are ignored in the model building process. Therefore, thermal errors of shape elements such as the spindle drift or the tilt motion can't be properly and explicitly incorporated into the volumetric error model. Hence, the equivalent vector approach mainly applied to modeling geometric errors of machine tools or coordinate measuring machines.

Since the separated element approach recognizes shape elements as with real dimensions, joint elements are defined as ball screws. Although thermal errors of shape elements are incorporated into a volumetric error model successfully, the volumetric error model derived by using the separated element approach is more complex than the Eq. (12) because of Abbe error terms expressed as multiplication of the rotational errors of joint elements by the real dimensions of shape elements. The reason why the Abbe error terms are intervened can be explained as follows: As shown in Fig. 3, local coordinate systems and ball screws are located at the outside of the workspace. The distance between the ball screw axis and the workspace can be represented as dimensional information of shape elements. Therefore, in order to represent the volumetric errors within the workspace with joint errors along ball screw axes, the Abbe error terms are additionally intervened. And, the derived volumetric error model requires dimensional information of machine tools for error compensation.

Since the virtual-shape approach recognizes shape elements as hypothetical elements that have only shape errors without real dimensions, all the above disadvantages of traditional approaches are improved by introducing the virtual-shape element.

5. Conclusions

A method for modeling volumetric errors of machine tools using the virtual-shape element is proposed. Following conclusions are obtained:

1. Traditional error synthesis methods can be classified into two approaches: the equivalent vector approach and the separated element approach. Both approaches have several conflicting good and bad points in model building and parameter identification processes.

2. In order to improve the problems of the previous modeling techniques, a concept of a virtual-shape element is proposed. The virtual-shape element is defined as a hypothetical element that has only shape errors without real dimensions and expressed as the virtual-shape transformation matrix.

3. A volumetric error model for the C-shaped vertical machining center is derived on the basis of the equivalent diagram that consists of joint elements and virtual-shape elements. The derived error model includes both geometric and dominant thermal error terms explicitly.

4. In order to incorporate thermal errors of shape elements into a volumetric error model, the Abbe error terms expressed as multiplication of the rotational errors of joint elements by the real dimensions of shape elements are unavoidable in the previous method. These errors make the model building process more complex. However, by introducing the virtual-shape element, the Abbe error terms do not exist in the derived volumetric error model.

5. Since three joint transformation matrices are redefined with the errors along three axes of the reference coordinate frame by introducing the virtual-shape element, the setup location of measuring instruments for parameter identification is coincided with the origin of the reference coordinate frame established at the vertex of the workspace. This solves the difficulties in the parameter identification process.

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