Kinematic and Stiffness Analysis of a Novel Ortho-Guided Tripod Machine Tool

Wei-Yao Hsu*and Jenq-Shyong Chen**
National Chung Cheng University, Department of Mechanical Engineering
* Ph.D. Student, **Professor

Abstract

This paper introduces a novel parallel kinematic machine (PKM) called Ortho-Guided Tripod (OGT) which is constrained to move in X, Y, and Z-directions by using an orthogonal guiding system. This paper analyzes the kinematic and stiffness of the OGT machine. It shows that the OGT machine has more consistence and higher stiffness over the workspace. The position of the OGT machine is directly measured along the unforced orthogonal guiding system which is separated from driving system, it means joint location and deformation, and thermal expansion of the OGT can be neglected, and the OGT has higher accuracy.

Keywords: Tripod, parallel kinematic machine

Introduction

The development of the parallel kinematic machines (PKMs) has been moved from six parallel-links to three parallel-links [1], for six parallel-links structure has some disadvantages: 1. poor ratio of workspace to machine size, 2. singularity problems, 3. complicated solutions of kinematic model, and 4. high cost of servo control system [2][3].

There are some commercial three parallel-links machines such as Tricept from Neos Robotics AB which is constrained by linear central tube and universal joint, Delta C300 form Hitachi, Ltd. which is constrained by the universal joints of one end of the links, and SKM400 form Heckert GmbH which is constrained by the rotary parallel 4-bar linkages.

We introduce a new kind of three parallel-links machines called Ortho-Guided Tripod (OGT) which is constrained by three orthogonal linear guides, as shown in the Fig. 1. OGT machine used ball joint of the link end attached moving platform and universal joint of the other end, and used the orthogonal linear guides, so that the OGT machine has three degree of freedoms. By using the orthogonal linear guide way system, we should put the position sensors on the guide ways, and the high precision of the moving platform can be expected.

It is well known that parallel kinematic machine has high rigidity due to its strut construction. But there are some different opinions [5] [6], for the components such as ball joints and universal joint used in the PKMs have poor rigidity. The static stiffness analysis of the parallel kinematic machine which can be represented by Jacobian matrix has been studied by many researchers [7] [8] [9]. We drive Jaocbian matrix from kinematic constraint equations of the new kind machine and evaluate the stiffness map.

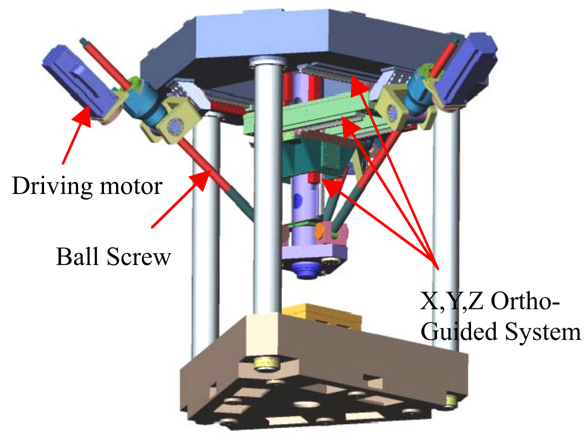

Fig. 1 Ortho-Guided Tripod Machine Tool

Inverse kinematic

The inverse kinematic problem is to find the limb lengths q_1, q_2, and q_3, when the moving coordinates X are given. That is

$q = Inv(\rho, X)$, where ρ is the kinematic parameters of tripod machine.

The inverse kinematic model can be obtained through vector loop analysis. As shown in fig. 2, the closed vector loop of each limb can be described as

$$B_i + Q_i = X + RP_i,\ i = 1 \sim 3 \qquad (1),$$

where,

$B_i = \begin{bmatrix} B_{ix} & B_{iy} & B_{iz} \end{bmatrix}^T$: base-joint location vector related to global coordinate,

$Q_i = \begin{bmatrix} Q_{ix} & Q_{iy} & Q_{iz} \end{bmatrix}^T$: leg vector related to global coordinate,

$X = \begin{bmatrix} x & y & z \end{bmatrix}^T$: platform position vector related to global coordinate,

$P_i = \begin{bmatrix} P_{ix} & P_{iy} & P_{iz} \end{bmatrix}^T$: platform-joint location vector related to moving platform,

R: rotation matrix of moving platform related to global coordinate,

q_i: limb length.

The length of each leg is determined as:

$$q_i = \|Q_i\| = \|X + RP_i - B_i\|, i = 1 \sim 3 \qquad (2).$$

Since the orientation of the platform is constraint by orthogonal three slides, the rotation matrix **R** in the equation (2) is an identity matrix.

Forward Kinematic

The forward kinematic problem is to determine the position of the moving platform, i.e. $X = for(\rho, q)$.

If we arrange the coordinates of the three base-joints and three platform-joints as P1 $\left(-r_p\sqrt{3}/2, -r_p/2, 0\right)$, P2 $\left(r_p\sqrt{3}/2, -r_p/2, 0\right)$, P3 $\left(0, r_p, 0\right)$, B1 $\left(-r_b\sqrt{3}/2, -r_b/2, 0\right)$, B2 $\left(r_b\sqrt{3}/2, -r_b/2, 0\right)$, and B3 $\left(0, r_b, 0\right)$, and substitute these coordinated into the Eq. *2*, the forward kinematic model can be determined as:

$$x = \frac{q_1^2 - q_2^2}{2\sqrt{3}(r_b - r_p)} \quad \text{(3a)},$$

$$y = \frac{q_1^2 + q_2^2 - 2q_3^2}{6(r_b - r_p)} \quad \text{(3b)},$$

$$z = -\left(q_3^2 - \left(\frac{q_1^2 - q_2^2}{2\sqrt{3}(r_b - r_p)}\right)^2 - \left(\frac{q_1^2 + q_2^2 - 2q_3^2}{6(r_b - r_p)} - (r_b - r_p)\right)^2\right)^{1/2} \quad \text{(3c)}.$$

Notice that the forward kinematic model of the innovative tripod machine is a closed form solution. The time-consuming and iterated numerical analysis of the forward kinematic solution is eliminated.

Stiffness Analysis

In this section we develop methods for representing static forces acting on a tripod machine tool and for transforming them to the Cartesian-coordinates system.

First, let the stiffness matrix of each actuate link be defined as:

$$\tau = K_s \cdot \Delta q \quad \text{(4)},$$

where $K_s = diag[k_{s1}, k_{s2}, k_{s3}]$: the stiffness matrix of the actuator space,

$\tau = [\tau_1 \;\; \tau_2 \;\; \tau_3]^T$: forces applied at the actuate link,

$\Delta q = [q_1 \;\; q_2 \;\; q_3]^T$: deformations of each link.

The deformations of the actuate links have the geometric relationship as follow:

$$\Delta q = J\Delta X \quad \text{(5)},$$

in which $J = \left[\frac{X + RP_1 - B_1}{\|X + RP_1 - B_1\|} \;\; \frac{X + RP_2 - B_2}{\|X + RP_2 - B_2\|} \;\; \frac{X + RP_3 - B_3}{\|X + RP_3 - B_3\|}\right]^T$ is found by differentiating Eq. 2.

Using the potential energy principle, we have the following relationship:

$$V_s = \frac{1}{2}\Delta X^T K_d \Delta X = \frac{1}{2}\Delta q^T K_s \Delta q \quad \text{(6)},$$

where K_d is the stiffness matrix of the tripod machine tool transformed to Cartesian-coordinates system. Substituting Eq.5 into Eq. 6, we have:

$$K_d = J^T K_s J \quad \text{(7)}.$$

Eq. 7 implies that the external force applied on the end-effector is related to its deformation by the stiffness matrix K_d. Assume each actuate link have the same stiffness. Then Eq. 7 can be rewritten as:

$$K_d = k_s J^T J \quad \text{(8)}.$$

In eq.8 k_s is the combination stiffness of each actuate link including the universal-joint, ballscrew/nut, and ball-joint. As an example of a workspace 400mm_400mm_300mm (X_Y_Z), Table 1 shows the computer simulation results of the stiffness of each type OGT machines for different link horizontal angles.

Table 1 Stiffness OGT machines

horizontal angle	Stiffness range Kxx/Kyy/Kzz (100N/μm)
45°	1.2~2.6/1.3~2.6/2.7~4.3
60°	0.6~1.4/0.7~1.5/4.5~5.8

Conclusion

In this paper, we present a new kind of Tripod machine that the moving platform is guided by the three orthogonal linear guide ways. By putting the position sensors on the guide way system, the volumetric error of OGT machine is determined by accuracy of the guide way system but no the link and joint errors. We also show that the stiffness of the Tripod machine is higher and more consistent over the workspace than the conventional serial machines.

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