Measurement and Calibration of High Accuracy Spherical Joints

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1. Introduction

High accuracy robotic manipulators are required for numerous industrial applications; such as inline inspection of assembly lines; laser cutting; and precision pick and place operations. Parallel kinematic machines [1] offer benefits, compared to serial manipulators, due to their high stiffness and inherently superior accuracy. Many parallel robot implementations utilize two and three degree-of-freedom joints to connect the robot arms, with universal joints commonly used for the two degree-of-freedom joints. Significant research has led to practical implementations of three degree-of-freedom joints based on a modified universal joint concept. Whereas this simplifies joint construction, it introduces singularities and mechanical tolerances that degrade the overall robot accuracy. The goal of this research is to develop a high-stiffness spherical joint capable of a large, singularity-free workspace with accuracy on the order of several micrometers.

Spherical joints have been developed based on the ball and socket concept, wherein a spherical ball sits in a hemispherical cavity. Three designs are implemented; based on point and surface contact mechanisms. The first joint (point contact) is achieved by kinematically mounting the ball on three smaller spheres embedded in the socket, as per ball bar mechanisms [2]. The second joint (rolling contact) uses ball transfers instead of fixed spheres to significantly reduce friction. The third prototype implements surface contact through replication of the ball with a teflon-laced epoxy, to achieve a high accuracy and low-friction ball to socket mating surface [3]. Each joint design incorporates a magnetic preload wherein the socket is populated with various configurations of permanent magnets that attract the solid steel ball.

Each spherical joint prototype is tested using a spherical kinematic test rig that allows the independent actuation of each of the joints’ degrees of freedom: roll (rotation about a nominal x-axis), pitch (y-axis rotation), and yaw (z-axis rotation). Each prototype is measured by affixing a tooling ball to a shaft attached to the ball and mounting the joint in the test rig, which itself is mounted in a coordinate measurement machine (CMM). The test rig actuates the joint to various positions in its working space and the CMM measures the position of the center of the tooling ball.

Joint accuracy is characterized using a geometric sphere-fit algorithm to fit the measurement data to a standard sphere through the calculation of the best-fit sphere center and radius. The final (sphere-fit) error is the deviation of each measurement position from the best-fit radius. Additionally, a kinematic parameter estimation algorithm has been developed to characterize the individual error parameters within the joint, such as the eccentricity of the axes of rotation. The passive joint is treated as a three degree-of-freedom mechanism and the error parameters determined through a numerical Jacobian-based technique [4]. As the joint prototypes have no angular feedback and the test rig is relatively inaccurate, joint angle positions are not accurately known and needed to be re-calculated for each measurement location. The calibration sequence involves the initial estimation of kinematic error parameters (based on the current joint angles) followed by the update of the joint angles (based on the updated error parameters and the measurement location). This “leap-frog” algorithm continues until both the error parameters and the joint angles converge; the final (calibrated) error is the difference between the predicted and the measured joint position.

Measurements of the rolling contact prototype joint indicate a maximum sphere-fit error of twelve micrometers ($12 \mu$m) and a calibrated maximum error of eight micrometers ($8 \mu$m). The error of the CMM is estimated to be two micrometers ($2 \mu$m).

2. Design of Prototype Joints

The spherical joint is required to have a singularity-free working range of 180° and accuracy of the order of several micrometers. Three prototypes have been implemented, each comprising an aluminum socket machined from 3.5in stock and a 2in diameter solid steel ball bearing (Figure 1). Separation between ball and socket is prevented by a magnetic preload force supplied by a ring of magnets about the circumference of the socket. Three primary contact mechanisms have been investigated: point, rolling and surface contact.
The point contact prototype locates the ball kinematically upon three spheres embedded within the socket. Given minimal Hertz deformation due to the relatively low preload forces, the primary source of error for this type of joint is the accuracy of the ball (typically 5 micron). The main problem with this design is the friction inherent in the point contact region. The rolling sphere point contact prototype seeks to alleviate this friction problem by utilizing rolling ball transfers mounted in the socket in place of the solid steel spheres. This dramatically reduces the friction but potentially increases the errors due to the additional influence of the ball transfer accuracy. The full surface contact prototype creates an accurate mounting surface through replication with a Teflon-laced epoxy. This provides a larger contact region to minimize contact stresses with minimal friction due to the Teflon contact surface. Despite the potential accuracy of such a joint, the friction is still significant.

(a) Fixed sphere point contact  
(b) Rolling sphere point contact  
(c) Replicated full-surface contact

Figure 1 – Spherical Joint Prototypes

3. Design of Spherical Kinematic Test Rig

The spherical kinematic test rig (Figure 2) allows the actuation of each of the spherical joint’s degrees of freedom for the purpose of measurement in a coordinate measurement machine (CMM). It comprises three concentric rings that are actuated by radio-control servomotors: the x-axis motor is mounted on the base; the y-axis motor is mounted on the x-axis ring; and the z-axis motor is mounted on the y-axis ring. The z-axis motor is connected to the spherical joint via a velcro clutch-plate and a tooling ball is mounted on the joint shaft to allow measurements by the CMM probe. The test rig stiffness is designed to be significantly less than the joint stiffness in order to avoid deformation of the joint during measurements. The test rig can be manually actuated with a radio control unit and automated via a commercially available controller. The test rig is mounted in the CMM and then run through a set of approximately 30 poses, distributed evenly about the joint workspace. For each pose, the tooling ball center is measured, and the set of joint poses and measured positions compiled for analysis.

(a) Test rig and spherical joint  
(b) Test rig mounted in CMM

Figure 2 - Spherical kinematic test rig
4. Modeling of Spherical Joint

Each spherical joint prototype is modeled as a simple three degree-of-freedom mechanism, with three orthogonal axes: roll (rotation about the joint x-axis); pitch (y-axis rotation); and yaw (z-axis rotation). This allows each of the joint prototypes to be modeled and measured in the same fashion and the final accuracy characteristics compared. Two key models have been developed; a sphere fit model and a kinematic model. The sphere fit model assumes all axes intersect at a common location and fits a sphere center and radius to the measurement data by minimizing the residual error between the predicted tooling ball location and the measured position (Figure 3). The remaining residual error represents the accuracy of the joint.

\[ \varepsilon_i = R - \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2} \]

\[ F(x_c, y_c, z_c, R) = \min \left( \sum \varepsilon_i^2 \right) \]

Figure 3 – Spherical joint sphere fit model

The kinematic model extends the sphere-fit model by introducing a nominal forward kinematic model between the measurement system and the tooling ball (Figure 4). Six frames are introduced to describe the joint center in relation to the CMM coordinate frame; the roll, pitch and yaw joint angles; and the tooling ball center in relation to the joint center. The spherical joint angles are used as input to the kinematic model to determine the position of the tooling ball center. In a similar error minimization process as the sphere-fit, the nominal forward kinematic model attempts to fit the joint center and tooling ball location (radius) to the measurement data.

Frame descriptions:
1. Tm — measurement system frame
2. Ts — socket frame
3. Tbr — ball roll frame (\( \theta_1 \))
4. Tbp — ball pitch frame (\( \theta_2 \))
5. Tby — ball yaw frame (\( \theta_3 \))
6. TCP — tooling ball frame

Nominal Forward Kinematics:
\[ X(x, y, z) = F(\theta_1, \theta_2, \theta_3) \]
\[ = Tm \cdot Ts \cdot Tbr \cdot Tbp \cdot Tby \cdot TCP \]

Figure 4 -- Spherical joint structural loop and kinematic model

The error forward kinematics introduces position and orientation errors between each of the nominal kinematic frames (Figure 5) in order to characterize the location and magnitude of the joint errors. A numerical Jacobian matrix is computed from a sequence of joint positions, this matrix may be singular if too many parameters are included in the error kinematics. Through a process of elimination, the offending parameters are removed until the Jacobian is numerically well defined. In addition to the joint center and tooling ball location, six parameters are found to be significant: three orientation and three position errors representing the position and orientation of the actual joint center. A linear regression solver, utilizing the singular value decomposition of the Jacobian matrix, is then used to determine the error parameters by comparing the error forward kinematics with the measured positions. This process is derived from robot calibration applications [5], however in this case the input joint angles are not well known due to the inaccuracies of the spherical kinematic test rig. Using a “leap-frog” approach the joint angles are updated using a numerical inverse kinematics algorithm and the Jacobian and error parameters recomputed until
convergence is achieved. If the error parameters are set to zero (only the joint center and radius are calibrated), then the error forward kinematics becomes equivalent to both the nominal kinematic and sphere fit models.

$$X(x, y, z) = F(\theta, \varepsilon)$$

Jacobian:

$$J = \left( \begin{array}{c}
\frac{\partial F_1(\theta)}{\partial \varepsilon_1} & \frac{\partial F_1(\theta)}{\partial \varepsilon_2} & \cdots & \frac{\partial F_1(\theta)}{\partial \varepsilon_m} \\
\frac{\partial F_2(\theta)}{\partial \varepsilon_1} & \frac{\partial F_2(\theta)}{\partial \varepsilon_2} & \cdots & \frac{\partial F_2(\theta)}{\partial \varepsilon_m} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial F_j(\theta)}{\partial \varepsilon_1} & \frac{\partial F_j(\theta)}{\partial \varepsilon_2} & \cdots & \frac{\partial F_j(\theta)}{\partial \varepsilon_m} \\
\frac{\partial F_M(\theta)}{\partial \varepsilon_1} & \frac{\partial F_M(\theta)}{\partial \varepsilon_2} & \cdots & \frac{\partial F_M(\theta)}{\partial \varepsilon_m}
\end{array} \right)$$

Singular Value Decomposition:

$$J = U \cdot S \cdot V^T$$

5. Measurement and Calibration

The rolling-sphere spherical joint is measured in a CMM with an accuracy of 2 micron in the working range of the test rig. The accuracy distribution is presented in Figure 6: the sphere fit model results in a maximum error of approximately 12 micrometers (µm); the kinematic model reduces this error to 8µm. The mean and standard deviation of both models is of the order of 3µm. This indicates that the uncalibrated joint is accurate to within 12µm within its working range, and this can be further improved to 8µm with the addition of the position and orientation error parameters.

6. Conclusions and Future Work

Three spherical joint prototypes have been developed for parallel robot applications. Each joint comprises an aluminum socket and a solid steel ball, preloaded by permanent magnets. The point contact prototype mounts the ball kinematically in the socket. The rolling contact joint significantly reduces friction by kinematically mounting the ball on rolling ball-transfers. The surface contact prototype uses replication to achieve a high-accuracy mating surface. A spherical kinematic test rig is used to actuate each joint to a predefined position, which is measured by a
coordinate measurement machine. The resultant uncalibrated accuracy of the rolling contact joint was 12 micron, which is reduced to 8 micron through calibration.

A fourth prototype is currently being investigated, based on a fluid contact mechanism. The replicated full-surface design is extended to include three orifices placed around the socket to create a thin film of air between the ball and the socket. This should significantly reduce the friction inherent in the replicated prototype while maintaining the high accuracy expected of a replicated joint. The residual error distribution remaining after the kinematic calibration has a large discrepancy between maximum and minimum residuals. This indicates that the simple kinematic error model insufficiently describes the physical phenomena underlying the measured joint. An improved model will be investigated with the goal of achieving a maximum error of the order of 2 micron.

7. References


Keywords: spherical joint, calibration, metrology, parallel kinematic machines