

KINEMATIC COUPLING INTERCHANGEABILITY

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I. INTRODUCTION

Traditional studies of kinematic couplings, such as those by Slocum, Mullenheld, and Poovey [1,2,3], have focused on the need for high interface repeatability; however, modular machines and instruments require rapid, accurate interchangeability. Interchangeability of a kinematic coupling is the tendency of the centroidal frame of the top half of the interface to return to the same position and orientation relative to the centroidal frames of different bottom halves when switched between them [4,5].

When a kinematic coupling is used to mount a machine or component, the mounting error arises from irregularities in the surface and preload conditions, manufacturing variation in the interface geometry, and environmental influences such as temperature changes. The translational and rotational components of these errors are reflected through the structural path of the machine by geometric transformations, giving the error contribution from the kinematic coupling at a point of measurement interest, such as the tool tip. The goal here is to model interchangeability and to determine if measurement of kinematic coupling contacts before interface mating can be used to decrease interchangeability error. The results are applied to kinematic coupling designs for the base and wrist of an industrial robot.

II. KINEMATIC COUPLING DESIGNS

A typical kinematic coupling mates a triangular configuration of three hemispheres on one interface plate to three “vee” grooves on another interface plate, thus enabling essentially exact constraint of the six-degrees of freedom between the two bodies by Hertzian surface contact at six small regions. The main caveat to traditional ball/groove couplings, where the sphere diameters are approximately the widths of the vee grooves to which they mount, is that their load capacity is limited to that of the small contact regions.

To achieve greater load capacity yet maintain repeatability, the “canoe ball” shape (named as such because it looks like the bottom of a canoe) evolved to include an integrated tooling ball for calibration as shown in Figure 1. The “canoe”, made by precision contour grinding, emulates the contact region of a ball as large as one meter in diameter in an element as small as twenty-five millimeters across.

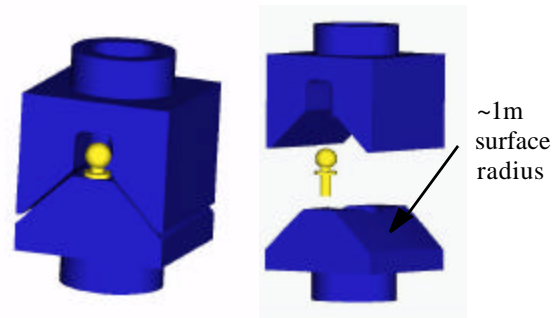


Figure 1: Canoe ball coupling elements.

III. INTERCHANGEABILITY MODEL

Neglecting the small variations in repeatability that may occur from relatively larger errors in the coupling geometry, a first-order estimate of the total mounting error for a kinematic coupling is the sum of the repeatability and the interchangeability errors. Repeatability of typical ball-groove and canoe ball kinematic couplings has been measured at one micron and better under well-controlled mounting conditions [1,2].

Interchangeability, on the other hand, is a deterministic geometric error. The kinematic behavior of a triangular coupling layout reduces the interchangeability error at the center of stiffness (the coupling centroid) to about one-third of the error of the coupling placements. The remaining error can be reduced by mapping the geometric errors based on the measured positions and orientations of each of the balls and grooves.

To model the interchangeability, first consider a general machine design application in which two

modules mate through an interface of ball-groove kinematic couplings, shown for an industrial robot in Figure 2. The grooves sit on a plate fixed to the floor and the mating balls are attached to the foot of the robot. Reference coordinate frames are placed centroidally on the groove set (A_{groove}) and the ball set (A_{ball}), and the couplings are secured using a sufficient (e.g. bolted) preload. The tool center point (TCP) and co-located coordinate frame (A_{TCP}) are offset from the ball coordinate frame by a translation and rotation described by the homogeneous transformation matrix (HTM), ${}^{TCP}T_{Ball}$. When the coupling balls and grooves are placed nominally, A_{ball} and A_{groove} are coincident.

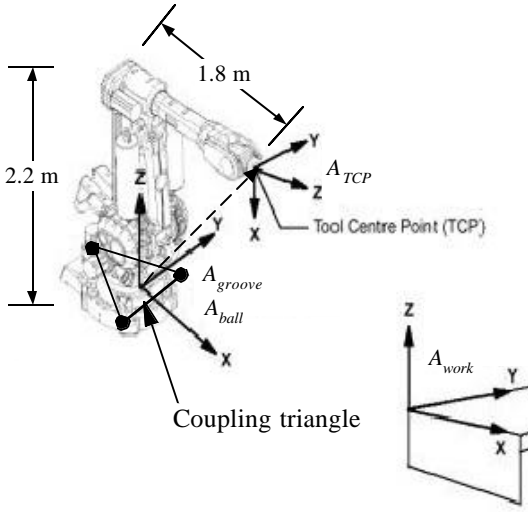


Figure 2: Frame nomenclature.

When error in the kinematic couplings is present, the total error at the TCP is the sum (average of worst-case and RSS) of the component errors at each of the contact points, expressed as a transformation matrix between the nominal and true centroidal frames of each interface half. These matrices, calculated using measurements of the contact points before the interface halves are mated, are denoted ${}^{Ball-true}T_{Ball-nom}$ and ${}^{Groove-true}T_{Groove-nom}$, and are specified to a model of the kinematic constraints between the balls and grooves. The output of the model, ${}^{Ball-true}T_{Groove-true}$, represents the mating error between the centroidal frames

The interface transformation, ${}^{Ball-nom}T_{Grv-nom}$, accounts for the total interchangeability error between the kinematic coupling balls and grooves:

$${}^{Ball-nom}T_{Grv-nom} = ({}^{Ball-true}T_{Ball-nom})^{-1} \dots$$

$$\dots \dots {}^{Ball-true}T_{Groove-true} \dots {}^{Groove-true}T_{Groove-nom} \quad (1)$$

This is added to the machine's forward kinematics to reduce the interchangeability error at the TCP, giving the residual error transformation at the TCP:

$${}^{TCP-corr}T_{TCP-nom} = (T_{interface} \cdot {}^{TCP-true}T_{Grv-nom})^{-1} \dots \dots {}^{Work}T_{Grv-nom} \quad (2)$$

IV. INTERCHANGEABILITY SOLUTION METHOD

When contact surfaces or offset features such as tooling balls of kinematic couplings are measured, the geometric mating relationship between the centroidal frames of the interface halves is found by solving a system of twenty-four linear equations. Specifically, measurement of the canoe ball interface gives location estimates for the following features of the balls and grooves, shown in Figure 3:

1. $[R_1, \dots, R_6]$: The radii of the six spherical contact surfaces.
2. $[\hat{q}_{S,1}, \dots, \hat{q}_{S,6}]$: Position vectors directed from each sphere center to the centroid of the ball interface.
3. $[b_1, \dots, b_6]$: The base points of the six groove flats, relative to the measurement frame (A_{MS}).
4. $[\hat{N}_1, \dots, \hat{N}_6]$: Normal vectors to the six groove flats, in A_{MS} .

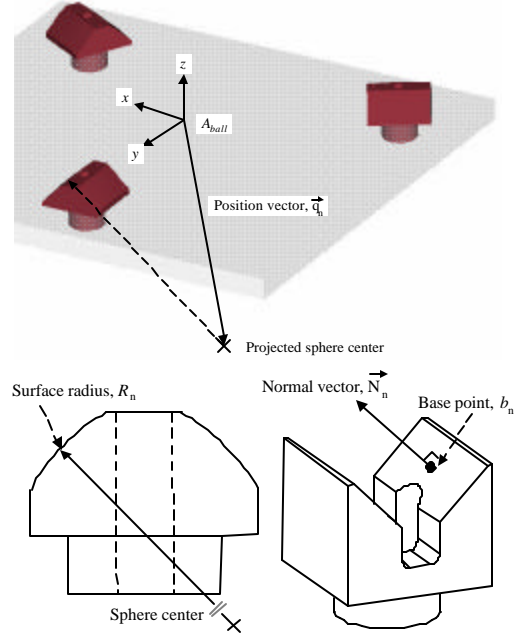


Figure 3: Canoe ball and groove measured features.

The eighteen unknown rest positions of the sphere centers are denoted $[p_{S,1}, \dots, p_{S,6}] = [(x_{S,1}, y_{S,1}, z_{S,1}), \dots, (x_{S,6}, y_{S,6}, z_{S,6})]$. The remaining six unknowns are the six error offsets between the centroidal frames of the interface plates, $[\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z, \mathbf{q}_x, \mathbf{q}_y, \mathbf{q}_z]$.

Separating variable and constant coefficients, the system has form $AX = B$, where X is the 24-element vector containing the unknown final positions of the spheres with respect to the measurement system and the six error motions of the interface. After inverting A and multiplying the result by B , the six error motions between the centroidal frames are elements of X . With small angle approximations, the transformation between centroidal frames is

$${}^{Ball-true}T_{Grv-true} = \begin{bmatrix} 1 & -\theta_{z_c} & \theta_{y_c} & \varepsilon_{x_c} \\ \theta_{z_c} & 1 & -\theta_{x_c} & \varepsilon_{y_c} \\ -\theta_{y_c} & \theta_{x_c} & 1 & \varepsilon_{z_c} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3)$$

To construct the system of equations, first consider that when the interface is seated, the projected center of each spherical surface will be as close as possible to its mating groove. Hence, the line passing through the projected center of the each sphere and the contact point between the sphere and its mating groove flat will be normal to the flat. Then, the distance between the projected sphere center and the groove flat is equal to the measured radius of the spherical surface. For example, the mathematical constraint between the first sphere and mating flat for the first canoe ball to groove pair, with unknown p_1 , is

$$\frac{(p_1 - b_1) \cdot \hat{\mathbf{N}}_1}{\|\hat{\mathbf{N}}_1\|} = R_1. \quad (4)$$

A group of six similar equations, one for each sphere/flat pair, contains the eighteen final coordinates of the sphere centers as unknowns.

Second, the measured distances between the sphere centers, calculated using $[q_{S,1}, \dots, q_{S,6}]$, must not change. The motions $[\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z, \mathbf{q}_x, \mathbf{q}_y, \mathbf{q}_z]$ of the centroidal frame of the ball interface (A_{ball}) with respect to the centroidal frame of the groove interface (A_{groove}), can be expressed in terms of the final positions of the sphere centers. In order to calculate the matrices A and B , small angle approximations must be made such that:

$$x_{S,1} = \delta_{x_c} + u_{S,1} - v_{S,1}\theta_{z_c} + w_{S,1}\theta_{y_c}, \quad (5)$$

$$y_{S,1} = \delta_{y_c} + u_{S,1}\theta_{z_c} + v_{S,1} - w_{S,1}\theta_{x_c}, \quad (6)$$

$$z_{S,1} = \delta_{z_c} - u_{S,1}\theta_{y_c} + v_{S,1}\theta_{x_c} + w_{S,1}. \quad (7)$$

Taken for the position of each sphere, these are the final eighteen equations of the system.

V. MODEL VALIDATION

The canoe ball interchangeability model was validated by building a series of small prototype models and measuring the error in the positions and orientations of their centroidal frames over all possible combinations of ball sets and groove sets. A large baseplate with two arrangements of six grooves at equal 60-degree angles around a center point, and ten smaller top pallets each with an equilateral canoe ball arrangement, were manufactured. To ensure statistical confidence in the calibration-interchangeability relationship, the locations of the coupling mounting and alignment hole pairs on each plate were intentionally perturbed within circular tolerances zones of 3-sigma diameter 0.64 mm from their nominal positions.

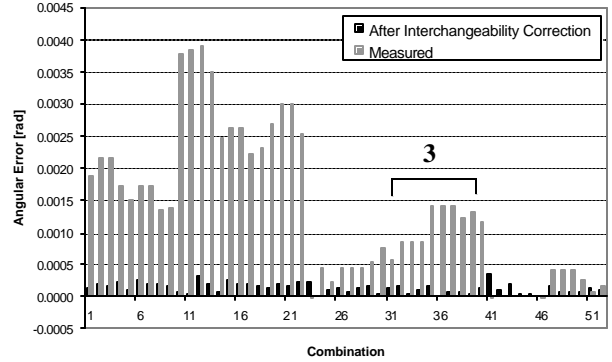


Figure 4: In-plane error of canoe ball pallets.

Figure 4 plots the in-plane angular error of each interface combination (choice of a pallet, groove set on the baseplate, and relative orientation), as measured between the centroidal frames, and after the transformation correction was applied. The combinations are grouped for each of the five pallets. The fifth pallet, for which the interchangeability correction actually increases the error for some trials, was machined with no more than 0.01 mm deviation from the nominal mounting hole locations. Over all trials, applying the interface transformation reduced the placement error by an average of 92%.

VI. APPLICATION TO INDUSTRIAL ROBOTS

Kinematic couplings were designed for the base and wrist interfaces of an ABB IRB6400R six-axis industrial robot manipulator, shown in Figure 5. The base is normally restrained with eight 20 mm diameter bolts. The new three-bolt base alternatives are a canoe ball interface, a three-pin interface (see [6, 7] for details), and a groove-cylinder interface (see [4] for details). The new wrist mountings used canoe ball couplings, or a three-pin coupling.

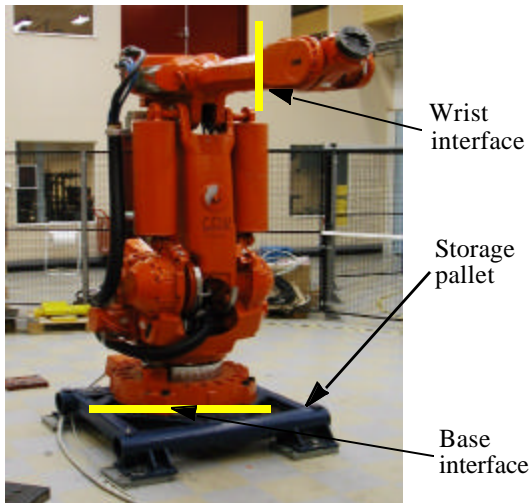


Figure 5: ABB IRB6400R industrial robot.

Interchangeability simulations specified the geometry of the manufactured prototypes and manufacturing and assembly tolerances representative of high volume production. 10,000 iterations were conducted for each level of calibration complexity (from no measurement, to measurement of two points per mount).

For the base interface, the interchangeability model predicts that the interface transformation accounts for approximately 50% or 0.11 mm of the 0.22 mm average total interchangeability error when full calibration is performed relative to offset measurement features. Similar results were obtained for the wrist interface. When the contact surfaces are measured directly, the interchangeability analysis reduces the tool point error by 88% to 0.02 mm. In the latter case, the remaining error is solely from measurement error; in the former case, variation in the dimension and placement of the measurement feature is also a factor. Negligible accuracy is gained by knowledge of the relative ori-

entations of the balls and grooves. Hence, unless the process of mounting the couplings to the plates is poorly controlled, only measurement of a single feature is needed for very good performance when offset measurement is performed.

VII. CONCLUSIONS

Direct measurement of the contact points on the halves of a kinematic interface can greatly reduce the effect of tolerance errors on mounting accuracy, with the residual interchangeability error based only on the error of the measurement procedure. In production, by making the contact surface measurements ahead of time, calculation of the interface transformation would be a step of the machine calibration routine. Ideally, the software would take the measurement values for the components, calculate the interface HTM, and apply it to the machine kinematics. The pre-measured placements of the contacts could be written to an identification tag on the interface, or the interface serial number could serve as a database key to the calibration data.

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