Scale-space and multi-resolution techniques for outlier analysis in surface metrology

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1. Introduction

Scale-space techniques and multi-resolution analysis are entirely different techniques, yet they share certain similarities. Scale-space approach relies on viewing a profile at different scales, much like the methodology used by wavelets. However, there is no clear transmission characteristic at each level. Both methods let us break down a signal into different scales and later reconstruct the signal. This permits us to analyze signal according to scale. The objective of this work is to demonstrate the applicability of scale-space technique and multi-resolution approach for outlier analysis in surface metrology.

2. Morphological Filters

Morphological filters [1] are a superset of the envelope filters used in surface metrology in the late 70s. They typically involve finding the contact between a structuring element (ball or line) and the profile. Two basic operations are dilation (locus of points of a ball rolling over a surface) and erosion (locus of points of a ball rolling under a surface). Closing and opening are combinations of these two fundamental operations. A closing filter produces an outer mean line and an opening filter produces a lower mean line. These mean lines can be used for identifying peaks and valleys, as they are not average mean lines such as those produced by conventional filters. Instead local peaks and valleys very heavily influence the morphological mean lines.

3. Alternating Symmetrical Filters (ASF)

ASF is a sequence of opening and closing filters [1] applied to a profile. ASF produces a ladder structure that is very similar to that produced by multi-resolution analysis using wavelets. At each step of the ladder, an approximation and a difference profile are created. The approximation is obtained by passing the approximation from the previous step through an opening filter of certain size followed by a closing filter of the same size. The sequence of closing and opening filter removes all peaks and valleys whose sizes are smaller than the size of the structuring element. The difference profile at any step is simply the difference between the approximation at the previous step and the approximation at that step. The next rung of the ladder is obtained by

![Figure 1 Schematic of alternating symmetrical filter](image)
using a structuring element of larger size. Figure 1 shows the schematic of alternating symmetrical filters.

Fig 2 (a) Original profile (b) final profile after valley removal (c) and the difference

Fig 3 Scale space analysis to remove outliers – the difference profiles with the 4 sigma threshold is plotted on the left, the approximation profiles on the right

Outlier suppression is performed by thresholding the difference profile at each scale and reconstructing the signal. An example profile shown in figure 2(a) is sent through an ASF and the resulting ladder structure is shown in figure 3. The structuring
elements considered are 0.008mm, 0.016mm, 0.032mm, 0.064mm, 0.128mm and 0.256mm. It can be seen that the valleys are mainly present in the second and third rung from the top. In this example, a global four-sigma threshold is applied to all levels. This, however, need not be the case. The thresholded difference profiles along with the final approximation are added to generate the modified profile as shown in the figure 2(b). The data points that are removed are shown in the figure 2(c).

4. Multi-resolution approach to outlier removal

Multi-resolution analysis [2] involves breaking down a signal into several scales. At each scale, an approximation and a detail are created that is very similar to the scale-space concept. One fundamental difference between the two techniques is that each scale of the multi-resolution ladder can be correlated to a certain cutoff frequency while each scale of the scale-space ladder is related to the size of the structuring element. Figure 4(a) shows a profile with deep valleys that are sent through 9 levels of the multi-resolution ladder using coif4 wavelets. The choice of the wavelet is based on the recommendation by Liu [3] in his dissertation. Level 1 corresponds to a cutoff of 0.004mm and each subsequent level is double the cutoff of the previous level. The different levels are shown in figure 5. Figure 4(b) shows the reconstructed profile and figure 4(c) shows the points that are discarded.

5. Conclusions

This paper has demonstrated the use of morphological filters and wavelets for outlier analysis. While multi-resolution analysis partitions a signal into many bands with very high computational speeds, scale-space approach is computationally expensive. Faster algorithms [4][5] are available for line structuring elements that enhance speed considerably. Scale-space seems to provide better valley separation with limited edge effects along the valleys.
6. References

4. Demin Wang, Dong-Chen He, “A fast implementation of 1-D grayscale morphological filters”, IEEE transactions on circuits and systems-II: Analog and Digital signal processing, vol 41, no.9, September 1994

Fig 5 Scale space analysis to remove outliers – the details are plotted on the left, the approximations on the right