Evaluation of Variations Specified by Non-Uniform Profile Tolerance Boundaries

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1. Introduction

Profile tolerance specification is a heavily utilized scheme to represent allowable geometric variation of manufactured parts as it simultaneously controls form, orientation and location of actual features of a manufactured part. The specification is applied on implicit features such as planes, straight lines and also on freeform curve and surface features. Examples of such freeform features include airfoils and bathtubs.

In our previous work [1], we showed that offsets (uniform boundaries) as defined by the ANSI or the ISO for profiles do not adequately define the tolerance zone as per the designer's intent in several cases. Furthermore, the applicability of non-periodic B-splines [2] was demonstrated to overcome the problems caused by offsets and to meet the needs for computer interpretable representation. Non-periodic B-splines can be used to specify non-uniform boundaries.

The deviation of the actual feature within a non-uniform or uniform zone is essential to map the relationship between parts and performance. Determination of deviation of actual features lying in certain regions of the tolerance zone and the parametric form of the curves renders this task a non-trivial problem.

In this paper we first group 2-D parts based on the type of the tolerance boundary. Later we explain deviation in the context of non-uniform tolerance boundaries followed by the application of two algorithms to determine deviations of actual features. Results from some example cases are also presented.

2. Profile Tolerance Boundaries

In industrial practice, a part is sub-divided into features and tolerances are specified to represent the extreme boundaries within which the feature should lie. Additional rules are provided on how adjacent features are to merge for example, chamfer and radius controls. These controls are provided to handle singular zones as described below.

Consider the hatched region in Figure 1. While any point in the rest of the tolerance zone can be associated with a point on the nominal geometry in the normal direction, the entire hatched region may be associated with the corner point of the nominal feature. Manufactured parts will not have distinct corners and hence extraction of co-ordinates of any actual point in this region is difficult. Furthermore, even if an actual point in this region is available, its deviation cannot be estimated, as it is difficult to associate the point with any particular feature. Such points are singular points and the associated zones are singular zones.

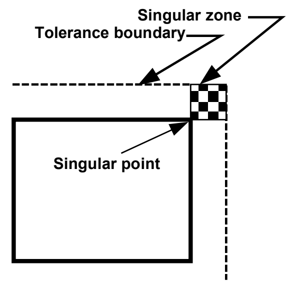

Figure1, *Singular zone within the tolerance boundary*

The occurrence of these instances can be distinguished. In part A of Figure 2, a corner point is shown on the nominal feature while in part B a point is shown on the tolerance boundary. These points on the curve do not posses a derivative. It is common practice not to include points in these zones in measuring deviations though resources are spent to control this region. While all parts with singular points on the boundary and on the nominal geometry can be grouped into a class, it is also possible to define parts without singular points and zones. Part C in Figure 2 is one such part and it has a tolerance boundary specified using B-spline that has a smooth transition between sections of varying tolerances. (Uniform boundaries need not have singular points and zones.)

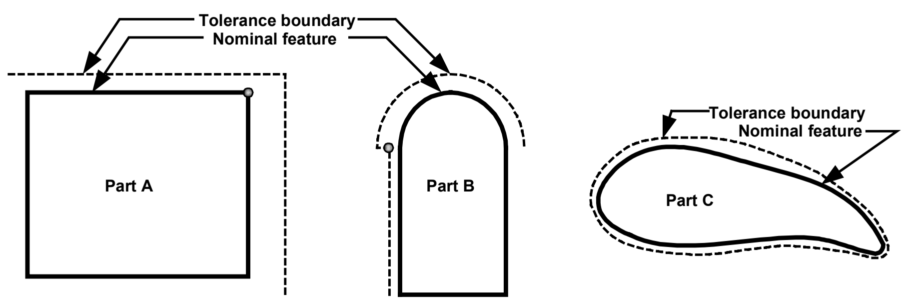

Figure2, *Parts A and B with uniform boundaries and singular points; Part C has non-uniform boundary without singularities*

Non-periodic B-splines can be used to specify profile tolerances for both classes of parts. While it is possible to arrive at the maximum deviation as per the conventional practice for parts with singularities, no single actual value can meaningfully characterize a part with non-uniform boundaries. The following section is a discussion on the deviation for non-uniform boundaries.

3. Deviation of the Actual Feature in *t*+ Boundary

The distance L (refer Figure 3a) between a point NP_i, on the nominal feature and a point AP_i of the actual surface that lies in the direction normal to nominal surface at NP_i is the deviation that can be inferred from the ANSI standard for uniform boundaries. Note that no unique normal exits at the singular points. Hence the occurrence of the maximum deviation within the restricted length of the profile feature is tested for conformance and reported.

In case of features that fall into the subgroup of smooth tolerance boundaries, as shown in Figure 3b, we do the following. The deviation of a measured point of the actual feature can be shown as a vector whose magnitude represents the ratio

$$| AP_i - NP_i | / |t+_i - NP_i | \qquad (1)$$

in the direction normal to the nominal feature with the initial point NP_i. The vectors are shown in Figure 3c. For conformance all the points of the actual feature should lie within the zone. The above discussion can be easily extended to bilateral tolerances. For deviations inside the nominal feature, deviation vectors will point in the direction of the *t*- zone boundary points.

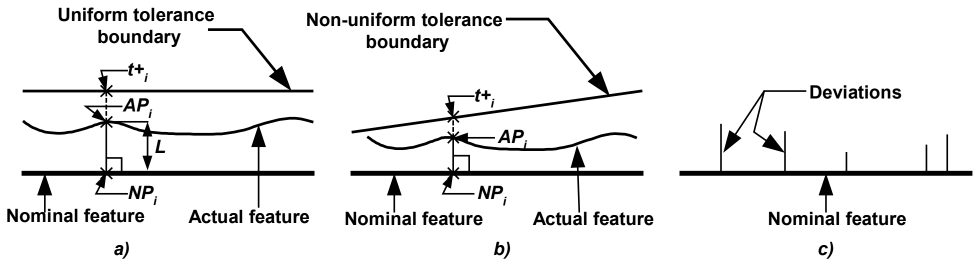

***Figure 3**, a) Deviation in the case of uniform tolerance boundary, b) Deviation in non-uniform tolerance boundary and c) presenting deviations for non-uniform boundaries*

4. Procedure for Evaluating Deviations

Classification of actual points with respect to tolerance boundary defined by B-splines i.e., if it lies within the tolerance zone or not, is not easy as in the case of implicitly defined curves. In the paper [1] for an actual point say a_i, we showed that if the closest point on the tolerance boundary b_i is known, we can check the containment of a_i in the zone.

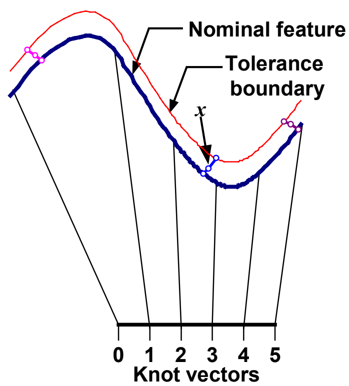

***Figure 4**. Determination of closest points on the tolerance boundary*

Refer Figure 4 where the nominal feature is a non-uniform B-spline with a knot vector that varies from 0 to 5. A linear scale shows how a parameter varying within the knot vector range maps to points on the nominal feature. In this example the tolerance boundary varies uniformly with respect to the nominal feature. Procedure_*profile*_evaluation below describes a method to determine the closest point and later we use it to calculate the deviations.

Procedure_*Profile*_evaluation

1. Set **range, delta_segment_length**. /* Range = a very small value, delta_segment_length = limiting condition */
2. Substitute each element in a knot vector set that does not repeat, $K_{sp} = [k_1,..k_j,..k_n]$, in the B-spline formulation of the boundary, to obtain a set of points $Kp = [(Px, Py)_1,.. (Px, Py)_j,...(Px, Py)_n]$ on the curve.
3. Of all the points in Kp, determine the point $(Px, Py)_j$ that is closest to the actual point.
4. If $j = 0$; choose the interval $[0, 1]$; else-if $j = n$, choose the interval $[n-1, n]$; else substitute (k_j - **range**) and (k_j + **range**) in the B-spline formulation to obtain two points on the curve. If the point corresponding to (k_j - **range**) is closer to the actual point than the point from (k_j + **range**), choose the interval $[j-1, j]$; else choose $[j, j+1]$. The case of point x in Figure 4 is an example of $j \neq 0,1$.
5. Application of Adaptive subdivision: Recursively subdivide the selected interval from step 4 by half and find the closest half, for example if $j = 0$, the first subdivided interval would be $[0,0.5, 1]$ or say $[P, Q, R]$. Substitute (Q - **range**) and (Q +**range**) in the B-spline formulation and test as in step 4 if the interval $[P, Q]$ or $[Q, R]$ is close. Say if $[P, Q]$ is closer, we subdivide the interval to obtain $[0,0.25,0.5]$ or $[P, P+((Q-P)/2), Q]$. Meanwhile, the points on the curve corresponding to P and R, i.e., $[U, V]$ on the curve is updated after every subdivision. This process is continued till the distance between U and V equals **delta_segment_length**. This method converges to the closest point quickly and paper [3] provides details on the performance of the adaptive subdivision.

Having obtained the closest point b_i close to a_i, the deviation can be calculated. In contact measurements, like co-ordinate measuring machines, the co-ordinates of the actual point is obtained with respect to the normal of the known point n_i on the nominal geometry. With a_i, b_i and n_i, the deviation can be calculated as described in the previous section.

Note that this method will work for actual points that are within the radius of curvature of the curve defining the tolerance boundary.

5. Results

The above algorithms can be used for both tolerance boundaries that are uniform and non-uniform. Figure 5a shows an implicit feature for which a uniform boundary is considered sufficient. The offset of end points of the feature can either be converted to an implicit form or a B-spline form of degree 1. For this example we used the theoretical inspection procedure (**Procedure_*Profile*_evaluation**) to determine the closest point on the tolerance boundary and later calculated the deviation. The result agrees with the deviation to be reported as per the ANSI standard. Figure 5b shows a higher degree nominal curve. In this case, using the **Procedure_*Profile*_evaluation**, closest points on both nominal and tolerance boundary was determined. Vectors as defined in section 3 are desirable to present the deviations for the example in Figure 5b. Figure 5c shows the deviations.

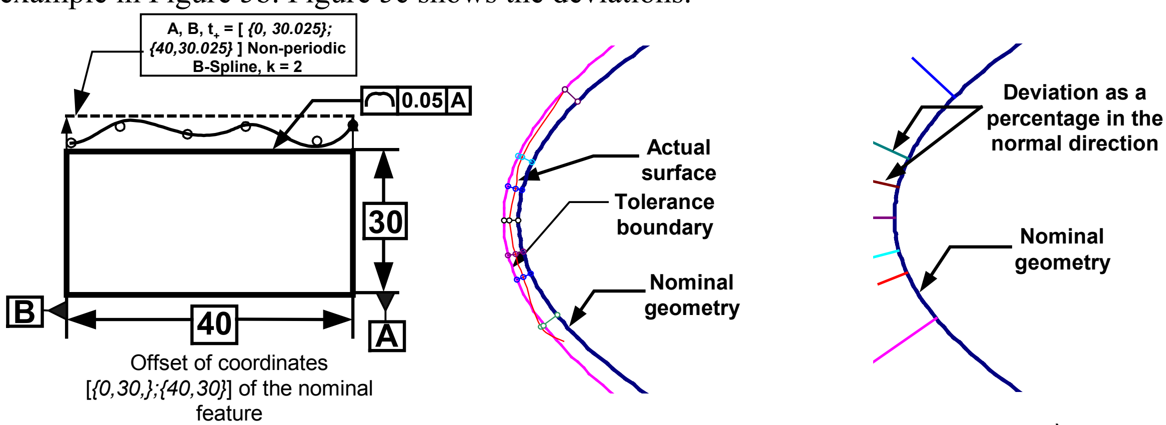

Figure 5, *a) Deviation for an implicit feature, b) Closest points on tolerance boundary and nominal geometry and c) Presentation of deviations*

6. Conclusions

Non-uniform B-splines can be used to represent uniform and non-uniform 3-D tolerance boundaries. A method to present deviations of non-uniform boundaries was illustrated. Two algorithms for inspection and evaluation of an actual feature were presented, as were results from the application of algorithms.

7. References

1. Kethara Pasupathy T M, Wilhelm R G, Curves for profile tolerance boundaries, 7th CIRP International seminar on computer aided tolerancing, ENS de Cachan, France, April 2001,pp.275-284.
2. Farin G E, Curves and surfaces for Computer aided geometric design, 3rd Ed, Academic Press.
3. Cohen E and Schumaker L, Rates of convergence of control polygons, Comp. Aid. Geom. Des., 2(1-3), 229-235,1985.