

# SYSTEMATIC FORM ERROR ESTIMATION FOR CIRCULAR PARTS USING FOURIER COMPONENTS

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**Abstract:** This paper presents a new approach to characterize the systematic form error in manufactured parts with circular features. A mathematical model using a combination of Fourier components is formulated to capture the unknown and significant systematic form error present in discrete coordinate measurement data collected from the manufactured surfaces. An objective measure, based on principles of statistical hypothesis testing, is established to identify the Fourier components to be included in this mathematical model. The effect of random form error on the accuracy of the estimation of systematic form error parameters is thoroughly investigated using simulated data.

**Keywords:** Systematic form error; Coordinate metrology; Fourier components

## Introduction

It is widely recognized that the actual form of manufactured features deviates from the ideal form specified in the CAD model. With existing manufacturing facilities, geometric errors are inevitable no matter how small they are. Therefore, a reliable inspection system is essential to evaluate the geometric errors precisely and then generate the necessary meaningful feedback information to improve faulty production processes and make accurate conformance decision against the required tight tolerances. A typical inspection system involves a device to take discrete coordinate measurement data as well as error analysis software to construct the geometry defined by these data points and compare with the design specifications to find out any discrepancy that may exist. To this end, the geometry constructed from the discrete data points has a significant influence in the correct identification of the geometric errors.

Most existing form error estimation methods are based on fitting an ideal substitute feature to the measured data points with an implicit assumption that manufacturing form error is totally random [1]. However, such assumption is impractical in most manufacturing cases, and very often leads to significant variation of inspection results [2,3]. For example, results obtained using various sampling distribution strategies are not identical, or different estimation techniques produce different results. This is basically because the correlation between errors in neighboring locations is not recognized and considered. Therefore, it is imperative to construct the surface profile that considers this correlation in order to reduce inspection uncertainty substantially. This requires characterizing the systematic form error due to reproducible imperfect production processes. However, it should be noted that manufacturing processes fundamentally consist of random error sources besides the reproducible ones [3]. Therefore, in parallel with characterizing the systematic form error, it is necessary to express the uncertainty due to unexplainable (random) form error.

This paper presents a new and systematic approach to characterize the systematic form error in manufactured parts with circular features and investigate the influence of the random form error component in the estimation of systematic form error parameters.

## Systematic form error representation

Since it is difficult to predict and consider all types of manufacturing errors in advance, a robust mathematical model is required to capture the unknown form error from the measurement data. In this work, a combination of Fourier components is selected to represent radially fluctuating systematic form error. Using the polar coordinate system, the deterministic profile,  $R(\theta)$ , is expressed as

$$R(\theta) = R + dR + SFC \quad (1)$$

where  $R$  and  $dR$  represent, respectively, the design radius and change in radius of the nominal circle during manufacturing. A sum of  $dR$  and the  $SFC$  term give the resultant systematic form error.  $SFC$  denotes fluctuations in the radial direction and is given by:

$$SFC = \sum_{i=1}^M a_i FC(m_i, \theta) \quad (2)$$

where  $M$  designates the number of significant Fourier components that are necessary to approximate the radial fluctuation errors.  $FC$  stands for either a sine or a cosine term.  $a_i$  and  $m_i$  represent the coefficient magnitude and spatial frequency of the corresponding Fourier component, respectively.

## Model determination

The deterministic profile model can be estimated by fitting the mathematical model given in Eq. (1) to the measurement discrete data points. However, the exact mathematical form of the model must be known before fitting proceeds; i.e. a specific and appropriate combination of Fourier components to be included in the model must be identified in advance. Such information is not available and also very difficult to determine. Therefore, the main focus of this research work is to develop a systematic approach that can identify the right combination of Fourier components to efficiently represent the fluctuating portion of the deterministic profile model. The major strategy adopted to accomplish this task is to first fit a candidate model with the simplest form to the data points. Then, the form of this candidate model is modified iteratively and refitted to the data points until a valid least squares fit is obtained. The form of the deterministic model can easily be modified by sequentially adding more Fourier components to the candidate deterministic profile model. The properties of the residual errors, such as correlation, distribution of residual errors around their mean, residual mean squares statistics, are examined to monitor the progress of the fitting process as more Fourier components are added to the model. Threshold values are set on these statistics to terminate the iterative fitting process based on principles of statistical hypothesis testing. The termination point indicates the adequacy of the candidate model to approximate the deterministic profile.

Since both form ( $FC$ ,  $a_i$ ,  $m_i$ ,  $M$ ,  $dR$ ) and location ( $x_o$ ,  $y_o$ ) parameters of a manufactured circular feature are not known in advance and must be estimated from the same data set, the estimates of location parameters will have an effect on the subsequent estimates of the form parameters, or vice versa. If a single numerical procedure is used to determine all these parameters simultaneously, the interaction between location and form parameters will create convergence problems. To overcome this problem, two separate optimization functions are formulated for finding the values of location and form parameters independently. These objective functions are expressed as

$$\min_{dR, a_i} \left( \sum_{i=1}^N [R_m^i - R(\theta)^i]^2 \right) \quad (3)$$

$$\min_{x_0, y_0} \left( \sum_{i=1}^N [R_m^i - R(\theta)^i]^2 \right) \quad (4)$$

where  $N$  represents the number of data points and  $R_m^i$  the radial distance from the origin of the local coordinate system  $(x_0, y_0)$  to the  $i$ th measured point.

Implementation of the proposed approach involves three important tasks to be carried out sequentially in each major iteration step. These tasks can be described as construction, localization, and validation of a candidate deterministic profile model. In a given iteration step, construction and localization of deterministic profile are first performed one after the other until the percentage error between the corresponding estimates obtained through consecutive repetition stabilizes. Thereby, the effect of correlation between location and form parameter estimates is considered. The model validation process then proceeds to check the randomness of the fitting residuals and hence test the adequacy of the candidate model in representing the deterministic profile. This is conducted using the serial correlation and chi-square ratio tests.

The flow chart given in Fig. 1 summarizes the overall procedure used to determine the appropriate deterministic model. The form error characterization process begins with preliminary evaluation of all candidate Fourier components. Then, the Fourier components are rearranged based on the magnitude of Fourier coefficients calculated in the preliminary evaluation process. In the last, but the most important, section of the algorithm, statistically significant Fourier components are determined with the serial correlation and chi-square ratio tests. Moreover, in a parallel and separate procedure to the determination of statistically significant Fourier components, the dominant Fourier components are identified using F-ratio test.

## Simulation studies

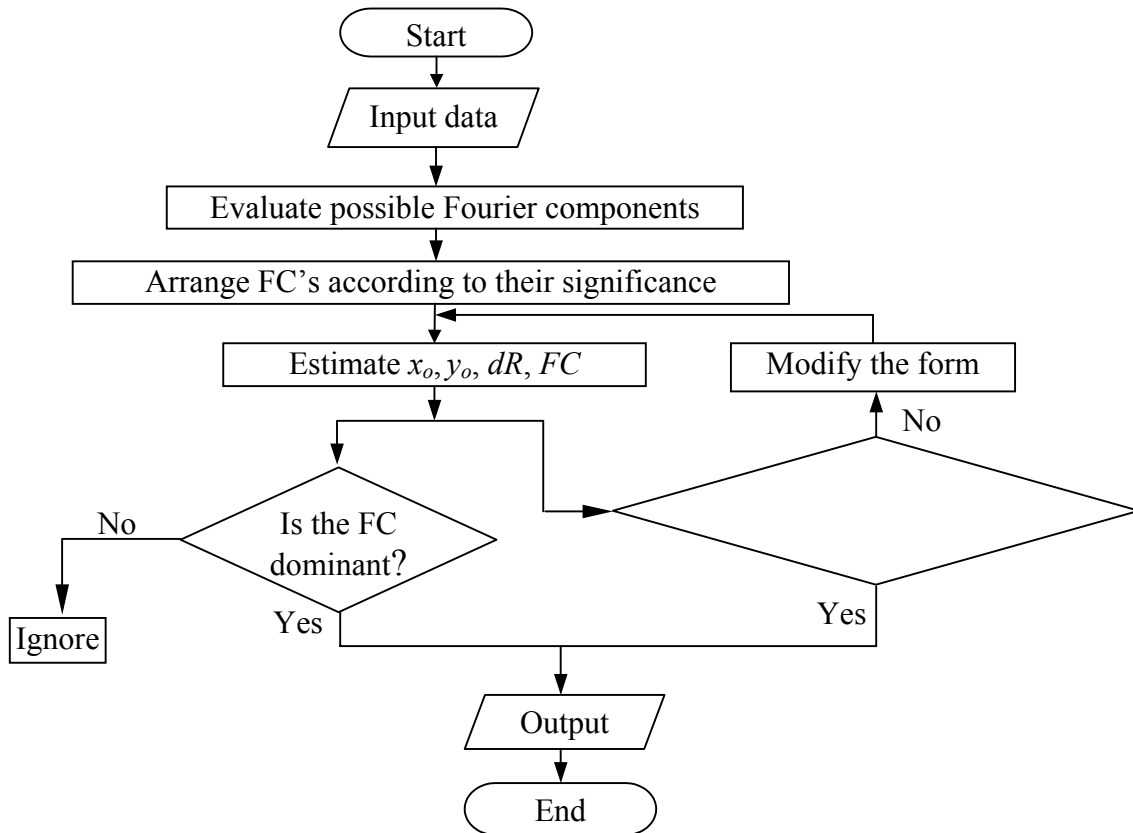
The proposed procedure has been implemented using several simulated point data sets. In order to generate the simulated data set, a hypothetical form error with specified characteristic is superimposed on an ideal circle. In this study, both periodic and non-periodic systematic form error combined with various proportions of random form error have been considered to investigate the effectiveness of this method in identifying the systematic form error accurately. Based on these simulation studies, the following observations have been made.

The proposed method reduces the influence of the systematic form error in the estimation of the center position of the circle. The center position estimated with the present method and the traditional least squares method has been compared. This comparison suggests that the estimated center position using this method has less uncertainty and is very close to the ideal position.

The proposed method can effectively characterize systematic form error when the “signal to noise ratio” is high and the form error is periodic. The performance of the procedure is fairly acceptable at lower signal to noise ratio and can also satisfactorily approximate non-periodic systematic form error. However, when the systematic form error is non-periodic and signal to noise ratio is very low, the coefficients of some of the statistically significant Fourier components become too variable and their values may considerably differ from their true values.

In such cases, it is recommended to construct the deterministic profile from the dominant Fourier components so that consistent and relatively reliable estimation is achieved.

The random form error influences the systematic form error characterization in two ways. It affects the values of the systematic form error parameters and it tends to introduce the wrong form parameters (Fourier components). However, the criteria used in this method satisfactorily prevent the latter incident. Furthermore, the effect of random form error depends on the signal to noise ratio used to simulate the data and the kind of systematic form error present.



**Figure 1** Procedure for determining deterministic profile model.

## Conclusions

In this paper, a systematic procedure is proposed to characterize the systematic form error in manufactured parts with circular features. The simulation studies suggest that this method performs well for a wide variety of form variations. Experimental verification is essential to illustrate the effectiveness of this method in practice. The related work is underway.

## References:

- [1] Yang, T. H. and J. Jackman, *ASME Journal of Manufacturing Science and Engineering*, 119, 375 (1997).
- [2] Dowling, M. M., P. M. Griffin, K. L. Tsui, and C. Zhou, *Technometrics*, 39(1), 3 (1997).
- [3] Yan, Z., Yang, B. D., and Menq, C. H., *International Journal of Machine Tools and Manufacture*, 39, 1199 (1999).