DERIVING THE PARAMETRIC ERRORS OF A CO-ORDINATE MEASURING MACHINE FROM THE MATHEMATICAL MODELS FITTED TO REPRESENT THE VOLUMETRIC ERROR COMPONENTS

JOAO BOSCO DE AQUINO SILVA e-mail: jbosco@ct.ufpb.br
FEDERAL UNIVERSITY OF PARAIBA
DEPARTMENT OF MECHANICAL ENGINEERING
58059-900 - João Pessoa –Pb -Brasil

Abstract

In this paper, an approach to volumetric determination of CMMs, which permits interpolation between measured points, has been developed. Mathematical and statistical techniques, such as Response Surface Methodology, have been used to represent the relationship between each volumetric error component (Ex, Ey and Ez) at a point, within the measuring volume, and the co-ordinates of the point (X_i, Y_i, Z_i). Also, a method to derive the parametric errors of a CMM from the volumetric error data is presented.

Keywords: Co-ordinate Measuring Machines, Parametric Errors, Response Surface Methodology

1. Introduction

This paper present a methodology to deriving the parametric errors of a co-ordinate measuring machines from the mathematical models fitted to represent the volumetric error components. These error components are obtained by measuring a novel form of space frame, which has been designed and manufactured, as described by (Silva (1997). This space frame has the form of a tetrahedron, which contains a sphere at each apex. The base of the tetrahedron comprises a ball plate that contains three spheres. Each tetrahedron contains three magnetic ball links. A simple magnetic ball link comprises a link, connecting magnetically, to two spheres. One sphere is located on the ball plate and the other at a space point where three links are connected together

2. Fitting mathematical models to represent the volumetric accuracy

Response Surface Methodology (RSM) has been applied to fit a mathematical model to represent the volumetric errors of CMMs. RSM, is a collection of mathematical and statistical techniques that are useful for the modelling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimise this response [1], [2]. By comparing the calibrated and measured tetrahedron configurations of the modular space frame it is possible to determine the volumetric error components (Ex_i,Ey_i,Ez_i) of the machine under test. The volumetric error at each point defined by the modular space frame is given as follows:

$$Ex_i = X_{mi} - X_i \quad (1)$$

$$Ey_i = Y_{mi} - Y_i \quad (2)$$

$$Ez_i = Z_{mi} - Z_i \quad (3)$$

where,

X_i,Y_i,Z_i, are the calibrated co-ordinates of the points generated by the calibrated modular space frame.

X_{mi},Y_{mi},Z_{mi}, are the measured co-ordinates of the points generated by the measured modular space frame.

The general equation that represents each volumetric error component (Ex, Ey, Ez) can be written either by using a first-order mathematical model, that is,

$$E_k(X_1, X_2, X_3) = b_{ko} + b_{k1}X_1 + b_{k2}X_2 + b_{k3}X_3 \quad (4)$$

or by using a second-order mathematical model,

that is,

$$\begin{aligned} E_k(X_1, X_2, X_3) = {} & b_{ko} + b_{k1}X_1 + b_{k2}X_2 + b_{k3}X_3 + b_{k4}X_1^2 + b_{k5}X_2^2 + b_{k6}X_3^2 \\ & + b_{k7}X_1X_2 + b_{k8}X_1X_3 + b_{k9}X_2X_3 \end{aligned} \quad (5)$$

where, k=x, y or z

In both equation (1) and (2) the coefficients β's are to be estimated by the method of least squares where the basic formula is given by the following equation:

$$b_k = (XX^T)^{-1} X^T Y_k \qquad (6)$$

where,
k=x, y or z and it is related to X, Y and Z direction , respectively.
$\mathbf{Y}_k$= the vector of error component Ex, Ey or Ez in X, Y or Z direction, respectively.
X^T= Transposed of matrix **X**
X= the matrix of independent or predictor variables X_1, X_2 and X_3.
X_1, X_2, X_3 = coded co-ordinates of the ith experimental point in the X, Y and Z direction, respectively.

2. **Deriving the parametric errors from the mathematical models fitted to represent the volumetric error components.**

Two cases have been considered to represent the volumetric error component in the X, Y and Z direction. Initially a first-order mathematical model was fitted. Next, a second-order mathematical model was developed. In both cases the residuals (Y_i - Y_{if}) were plotted against the fitted value, Y_{if}, obtained from the fitted mathematical model. By analysing the residual plots as shown in Silva (1999), it was found that the second-order model more adequately represents the measured data. The analysis of variance concerning the second-order mathematical model was established and it was observed that the overall regression is statistically significant. Therefore, the second-order mathematical model has been selected to represent the volumetric error component in the X, Y and Z direction.

Once the mathematical models that represent the volumetric error components (Ex, Ey, Ez) of a CMM has been fitted, it is possible to derived the parametric errors of the machine under test. To achieve this objective a particular method has been developed and applied as part of this research. This method is founded on the physical meaning of each parametric error component. Basically, the following steps must be performed when applying the proposed method. First, a reference plane within the measuring volume of the machine is selected. Second, measurement lines on the reference plane are defined. These measuring lines are defined by taking into account the physical meaning of each parametric error to be determined. Third, the boundary conditions are applied on the fitted equations that represent the volumetric error components(Ex, Ey, Ez). The parametric errors can be derived as following:

- Positioning error in X direction, $\delta_x(X)$, at plane Z=0.
  Boundary conditions: Z=0, Y=0, 0<=X<=X_{max}, Then,

$$d_X(X)\Big|_{Z=0;Y=0} = Ex(X, Y=0, Z=0) \qquad (7)$$

- Positioning error in Y direction, $\delta_y(Y)$, at plane Z=0.
  Boundary conditions: Z=0, X=0, 0<=Y<=Y_{max}, Then,

$$d_Y(Y)\Big|_{Z=0;X=0} = Ey(X=0, Y, Z=0) \qquad (8)$$

- Positioning error in Z direction, $\delta_z(Z)$, at plane X=0.
  Boundary conditions: X=0, Y=0, 0<=Z<=Z_{max}, Then,

$$d_Z(Z)\Big|_{X=0;Y=0} = Ez(X=0, Y=0, Z) \qquad (9)$$

b) Straightness errors

- Straightness error in the X direction when moving along the Y axis, $\delta_x(Y)$, at plane Z=0.
  Boundary conditions: Z=0, X=0, 0<=Y<=Y_{max} Then,

$$d_X(Y)\Big|_{Z=0;X=0} = Ex(X=0, Y, Z=0) \qquad (10)$$

- Straightness error in the direction Y when moving along the X axis, $\delta_Y(X)$, at plane Z=0.
  Boundary conditions: Z=0, Y=0, 0<=X<=X_{max} , Then,

$$d_Y(X)\Big|_{Z=0;Y=0} = Ey(X, Y=0, Z=0) \qquad (11)$$

- Straightness error in the direction X when moving along the Z axis, $\delta_X(Z)$, at plane Y=0.
  Boundary conditions: X=0, Y=0, 0<=Z<=Z_{max} ,Then,

$$d_X(Z)\Big|_{Y=0;X=0} = Ex(X=0, Y=0, Z) \quad (12)$$

- Straightness error in the Y direction when moving along the Z axis, $\delta_Y(Z)$, at plane X=0.
Boundary conditions: X=0, Z=0, 0<=Y<=Y_{max} , then,

$$d_Y(Z)\Big|_{X\,0;Y=0} = Ey(X=0, Y=0, Z) \quad (13)$$

- Straightness error in the direction Z when moving along the X axis, $\delta_Z(X)$, at plane Y=0.
Boundary conditions: Y=0, Z=0, 0<=X<=X_{max} , then,

$$d_Z(X)\Big|_{Y\,0;Z=0} = Ez(X, Y=0, Z=0) \quad (14)$$

- Straightness error in the direction Z when moving along the Y axis, $\delta_Z(Y)$, at plane X=0.
Boundary conditions: X=0, Z=0, 0<=Y<=Y_{max} , then,

$$d_Z(Y)\Big|_{X=0;Z=0} = E_Z(X=0, Y, Z=0) \quad (15)$$

c) Angular errors

- Pitch and Yaw errors

In this research, a method to determine pitch and yaw errors has been applied. This method consists in deriving the positioning error along two measuring lines that are parallel to the axis of motion. A distance, h, in the appropriated orthogonal distance, separates these measuring lines. Pitch and yaw errors can be calculated by using the following equations:

- Pitch error in X axis

$$E_Y(X) = \frac{d_X(X)\Big|_{Z=h,Y=0} - d_X(X)\Big|_{Z=0,Y=0}}{h} = \frac{Ex(X, Y=0, Z=h) - Ex(X, Y=0, Z=0)}{h} \quad (16)$$

Pitch errors in Y and Z axes can be determined following similar procedure.

- Yaw error in X axis

$$E_Z(X) = \frac{d_X(X)\Big|_{Y=h,Z=0} - d_X(X)\Big|_{Y=0,Z=0}}{h} = \frac{Ex(X, Y=h, Z=0) - Ex(X, Y=0, Z=0)}{h} \quad (17)$$

Yaw error in Y and Z-axes can be determined following similar procedure.

- Roll error in X axis

The roll error of the X axis can be derived by considering the straightness error, $\delta_Y(X)$, on two parallel planes, which are separated by an orthogonal distance, h. That error can be calculated by the following equation:

$$E_X(X) = \frac{d_Y(X)\big|_{Z=h,Y=0} - d_y(X)\big|_{Z=0,Y=0}}{h} = \frac{Ey(X, Y=0, Z=h) - Ey(X, Y=0, Z=0)}{h} \quad (18)$$

Roll errors in Y and Z axes can be determined following similar procedure.

d) Squareness errors between axes of motion

In this research, a method to determine the squareness errors α, β_1 and β_2 in the planes XY, XZ and YZ, respectively, has been proposed. Basically, this method uses the straightness errors, which are obtained from the mathematical models fitted to represent the volumetric error components. To describe this method, let us consider the straightness errors $\delta_x(Y)$ and $\delta_y(X)$ at plane XY. The slope of the best fit least square line to the curves defined by $\delta_y(X)$ and $\delta_x(Y)$ are θ_{yx} and θ_{xy} , respectively. The squareness error α is given by the following equation:

$$\alpha=\theta_{yx}-\theta_{xy} \quad (19)$$

Similarly, by applying the same method the squareness errors β_1 and β_2 can be determined. These errors are defined as follows:

$$\beta_1=\theta_{xz}-\theta_{zx} \quad (20)$$

$$\beta_2=\theta_{yz}-\theta_{zy} \quad (21)$$

where,

θ_{xz},θ_{zx},θ_{yz} and θ_{zy} are the slopes of the best fit least squares line to the curves defined by $\delta_x(Z)$, $\delta_z(X)$, $\delta_y(Z)$ and $\delta_z(Y)$, respectively.

3. Conclusions

In using mathematical and statistical techniques, such as response surface methodology, it can be concluded that a second-order mathematical model more adequately represents the volumetric error data obtained by measuring the modular space frame. The developed method to derive the parametric errors from the volumetric error data constitutes an efficient and rapid tool in diagnosing the sources of error of a CMM.

The technique developed for volumetric error calibration does not have to assume that the CMM under investigation has to behave as a rigid body kinematic system.

4. References

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