SYSTEMATIC ERROR ESTIMATION BASED ON GREY RELATIONAL ANALYSIS

Z. Wang *, Y. Gao **, S. Tse **, and P. Qin ***

* Luoyang Institute of Technology, Luoyang, 471039, China
** Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong
Email: meygao@ust.hk
*** Xi'an Jiaotong University, Xi'an, 710049,China

Introduction

It is known that systematic errors could cause the measurement results deviating from the true values. This could decrease the validity of the measurement. Systematic errors are due to the adverse effects in the measurement methods, apparatus, environments, etc. In measurement practices, due to the complexity of error sources, systematic errors are found difficult to determine or sometimes ignored under the assumption that such errors do not exist. In terms of solutions, simply increasing the number of repeated measurement, which is common in a traditional test, may not be able to reduce or eliminate the systematic error. Based on data distribution, the statistical criteria such as the experimental contrast, the residual error check-up, and the t-distribution inspection are often used to estimate the systematic errors in a measurement. The working principle is based on the statistics principle that the existence of systematic error in a measurement sequence would destroy the quality of randomness. To improve accuracy in a modern test, it is very important to examine the characteristics of the systematic errors and find an effective method to reduce their effects on the measurement.

In recent years, there are several investigations on the systematic error estimation and removal [1-5]. Rothe [6] introduced a multivariate estimation method to determine the systematic error of a surface profile using a covariance matrix. Bechhoefer [7] proposed a method to estimate the random error that exists in the experimental data along with the relative proportion of systematic error using an auto-covariance function of the residual errors in a number of least-square curve fittings. Kojro [8] discussed a number of digital statistical correlations and algorithms with respect to the systematic errors. The standard deviation of the statistical errors in the correlation estimators was also computed. Kadis [9] investigated the criteria to see if the random and systematic components of the total measurement error are sufficiently small to neglect. The method is within the framework of an approach to estimate the measurement errors. Rekkas [10] presented an algorithm to deal with the problem of systematic errors in radar measurement. The performance of the algorithm was assessed in several cases and the result was satisfactory. The above research work and the corresponding estimation methods are based on the statistics principle. To do a statistical analysis with sufficient confidence, the measurement number must be large enough, so that the distribution could be confirmed. In addition, to simplify the numerical computation, and in many cases to permit an analytical solution, the distributions are required to conform to some typical distributions such as the Gauss distribution and the t-distribution. In practical measurement, in particular in the tests where the data samples are only a few, such as in the destructive tests and in the blast-off examination, it is very difficult to obtain a large number of data samples. On the other hand, if the sample size of the measurement data is large, but it is not sure if it fits a typical statistical distribution, the statistical criteria may still be difficult to use. The result may not be true if the statistical methods are used to analyze the measurement data.

To solve this problem, a new method using the grey system theory is proposed to estimate the systematic errors in a dynamic measurement. In this method, a grey relational analysis is performed for a measurement sequence. Due to the use of the grey relational analysis, the requirements of large samples and conformance to typical distributions are no longer needed. As such, the systematic errors can be successfully determined for different measurement sequences without the stringent requirements. Theoretical analysis and experimental investigation are presented. It shows that the proposed method can be used to estimate systematic errors with high confidence where the statistical methods are found not suitable to use. The proposed method can be seen as a viable supplement to the statistical methods.

Working Principle

Measurement errors from apparatus, environmental factors and so on, make measurement results uncertain in certain degrees. Measurement systems could be regarded as grey systems. Measurement processes could also be regarded as grey processes [11]. The data would be correlated in certain degrees in a repeated measurement if systematic errors do not exist in the measurement sequence. Thus a grey relational analysis can be applied. The essence is comparing the geometric curves shape among sequences to get the quantitative description with grey relational coefficient and grey relational grade [12]. Assume the measurement is carried out using several different methods, where each method is done with repeated measurements. The sequence of a specific method can be expressed as:

$$x_i = x + \delta_i + \theta_i \tag{1}$$

where i=1,2,…,n, x_i is the sequence of the measured values, i is the sequence number, x is the true value sequence of the measurand, δ_i and θ_i is the sequence of random error and systematic error, respectively. If the referenced sequence is

$$x_0 = x + \delta_0 + \theta_0 \tag{2}$$

where x_0 is the referenced sequence of the measured value, δ_0 and θ_0 is the referenced sequence of random error and the systematic error, respectively. To analyze the systematic error, a pretreatment is to be done. The absolute difference between the measured sequence and the referenced sequence is expressed as:

$$\Delta_i = | x_i - x_0 | = |\delta_i + \theta_i - (\delta_0 + \theta_0)| \tag{3}$$

The grey relational coefficient between the two sequences can be defined as:

$$\gamma(x_i, x_0) = \frac{\min \ \Delta_i + \zeta \ \max \ \Delta_i}{\Delta_i + \zeta \ \max \ \Delta_i} \tag{4}$$

The grey relational grade [12] can be obtained from the equations. Since the absolute difference Δ_i is related to both the random and systematic error (Eq. (3)), the grey relational grade would be also influenced by both random and systematic error. The random error would be much less than the systematic error if it could be reduced into neglected degree by many-repeated measurement. The relational grade would mainly depend on the systematic error if the random error were able to be neglected. A specific type of systematic error in the measuring process would change the grey relational grades in the sequence. The grey estimation of systematic errors can be done using the grey relational grade and grey relational coefficient.

Measurement Results and Discussion

The measurement results of an experimental testing are examined to demonstrate the use of the method for the systematic error estimation. In the testing, a workpiece was measured repeatedly for a number of times using two different methods. The measuring results were as: $y_1(k)$ = [0.27, -0.78, 0.11, -0.71, 0.03, 0.07, 0.08, 0.56] (μm) and $y_2(k)$ = [-0.90, 0.20, 0.96, 0.86, -0.06, -0.90, -0.17, -0.03] (μm). A pretreatment mapping of the original sequences was used as:

$$\begin{aligned} x_i(1) &= (2y_i(1) + y_i(2))/3 \\ x_i(k) &= (y_i(k-1) + y_i(k) + y_i(k+1))/3 \\ x_i(8) &= (y_i(7) + 2y_i(8))/3 \end{aligned} \tag{5}$$

After above transformation, two new data sequences were obtained as $x_1(k)$=[-0.08, -0.13, -0.46, -0.19, -0.20, 0.06, 0.23, 0.40] (μm) and $x_2(k)$=[-0.53, 0.08, 0.67, 0.58, -0.03, -0.37, -0.36, -0.07] (μm). The smallest data point in the above sequence was assumed to compose the referenced sequence as $x_0(k)$ = -0.53, k=1,2,…,8. The grey relational coefficient between the measured sequence and the referenced sequence [12] was obtained as:

$$\gamma(x_i(k), x_0) = \frac{\min_i \min_k |x_i(k) - x_0| + \zeta \max_i \max_k |x_i(k) - x_0|}{|x_i(k) - x_0| + \zeta \max_i \max_k |x_i(k) - x_0|} \tag{6}$$

where ζ is a coefficient, and $\zeta \in [0,1]$. In general, ζ=0.5. The grey relational coefficients between the data of the measured sequence and that in the referenced sequence can be obtained. The grey relational coefficient curves are as shown in Fig. 1. The grey relational grades between the measured sequence and the referenced sequence are given as:

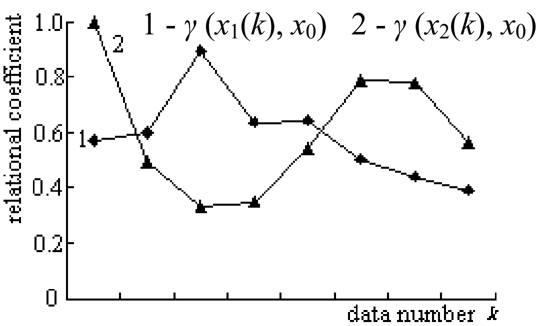

Fig. 1 Grey relational coefficient curves

$$\gamma_i = \frac{1}{8}\sum_{k=1}^{8} \gamma(x_i(k), x_0) \tag{7}$$

For the two sequences, γ_i=[0.586,0.608]. From Fig. 1, it can be seen that there were no linear or periodical features in the two curves. From Eq. (7), the difference between two relational grades is very small. In addition, there was no constant systematic error since the two measurement sequences were obtained using two different methods. Therefore, it can be said that there was no systematic error in the measurement process. The conclusion was the same, if the statistic criteria were used to estimate the systematic errors in the measurement process. Additional experiments show that if the difference is less than 0.4, there would be no remarkable systematic errors in a measurement process. The method can be served as an alternative method to estimate systematic errors for the cases where the statistical methods cannot be used due to unknown distribution and very small number of samples.

Conclusions

A new method is proposed to estimation the systematic errors in a measurement process. The method is based on a grey relational analysis of a measurement sequence using the grey system theory. Multiple data sequences can be analyzed. The proposed grey relational estimation does not require the measurement data to conform to the normal distribution and does not require a large sample size. It is therefore a good alternative to estimate systematic errors where the statistical methods could not be successfully applied.

Acknowledgement

This project is supported by the National Natural Science Foundation of China (Project No. 59805007).

References

[1] Abdou, I. E., Comment on the paper ’Elimination of systematic error in subpixel accuracy centroid estimation’, *Optical Engineering*, 1995, **34**(11), pp. 3347-3348, USA.

[2] Agupov, V. A., and Lotonov, H. A., Estimate of the systematic error of dosimetric measuring instruments in the absence of reference standards, *Izmeritel'naya Tekhnika*, 1998, **41**(8), pp. 50-52, Russia.

[3] Tao, R., and Tong, G., A new method for testing systematic errors caused by multifactors, *Chinese Journal of Scientific Instruments*, 1999, **20**(6), pp. 564-567, China.

[4] Liu, J., Owe, Fruhauf, and Andreas, Schonecker, Accuracy improvement of impedance measurements by using the self-calibration method, *Measurement*, 1999, **25**(3), pp. 213-225.

[5] Wang, J., and Fiebig, M., Measurement of the thermal diffusivity of liquids with a laser-induced thermal-grating technique, an experimental investigation of systematic errors, *International Journal of Thermophysics*, 1996, **17**(6), pp. 1229-1239, USA.

[6] Rothe, Hendrik, Multivariate estimation of the systematic error of scanning probes microscopes, *Thin Solid Films*, *Proc the Workshop on Development and Industrial Application of Scanning Microscopes*, SXM-1, 1995, **264**(2), pp. 282-290, Germany.

[7] Bechhoefer, J., Curve fits in the presence of random and systematic error, *American Journal of Physics*, 2000, **68**(5), pp. 424-429, USA.

[8] Kojro, Z., Riede, A., Schubert, M., and Grill, W., Systematic and statistical errors in correlation estimators obtained from various digital correlators, *Review of Scientific Instruments*, 1999, **70**(12), pp. 4487-4496, USA.

[9] Kadis, R. L., Criteria for negligible smallness of random and systematic components of measurement errors, *Izmeritel'naya Tekhnika*, 1996, **39**(7), pp. 3-7, Russia.

[10] Rekkas, C. M., Kyriakopoulos, K., and Krikelis, N. J., An algorithm for the estimation of systematic errors in radar measurements, *International Journal of Systems Science*, 1994, **25**(6), pp. 1105-1112, UK.

[11] Wang, Z., Qin, P., and Gao, Y., Grey evaluation of measurement uncertainty, *Journal of Grey System*, 1999, **11**(4), pp. 347-352, UK.

[12] Deng, J., Grey systems, China Ocean Press, Beijing, 1988.