Wavelet Analysis with Different Wavelet Bases for Engineering Surfaces

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1. Introduction

Wavelets are mathematical functions that separate data into different frequency components, and then present each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology. In recent years, some efforts have been taken to apply wavelet analysis to surface profile analysis [1][2][3]. The goal of this paper is to address the difference between wavelet basis in terms of transmission characteristics associated with their filter bank.

2. Wavelet Analysis Overview

The idea of approximation using superposition of functions has existed since the early 1800's, when Joseph Fourier discovered that he could superpose sines and cosines to represent other functions. However, the sines and cosines that comprise the bases of Fourier analysis are non-local functions. Therefore, frequency information obtained by Fourier transform is the average of the total spatial domain and the sharp change in the spatial domain will be lost due to the averaging. It is ruled by the Heisenberg uncertainty principle that multiplication of the temporal variance and the frequency variance will always be greater than 1/2. Therefore a tradeoff exists between temporal (spatial in surface analysis) localization and frequency localization of a basis function. Wavelets overcome the shortcoming of sines and cosines by having compact support in both spatial and frequency domain. And by varying the size of the support in both domains you can get different emphasis in the representation of spatial domain information or frequency domain information.

Multiresolution analysis in digital signal processing (DSP) is a major application with wavelets. It is introduced by Mallat and Meyer after discovering the unexpected equivalence between pyramid algorithm used in discrete multi-rate filter banks and wavelet transform [4][5]. While the pyramid algorithm enables fast implementation of multiresolution analysis, the wavelet theory builds strong mathematical ground for it. The details of the relation between wavelets and filter banks are given in the book written by Strang/Nguyen [6].

3. Wavelet Bases

Many wavelet families with various characteristics are known. Some wavelet families are especially useful for specific application. Most wavelets are based on FIR filters although research work is also done for wavelets constructed by IIR filters. The orthogonal and biorthogonal wavelets are the two basic categories of wavelets. Table 1 summarizes the main properties of most well-known wavelets.

The primary consideration in the use of wavelets for surface profile analysis is the amplitude and phase transmission characteristics of the wavelet basis. A combination of good amplitude and linear phase transmission is always desired to achieve minimum distortion of surface features. Among the listed wavelets in Table 1, Haar wavelet is the oldest and simplest wavelet that is not continuous [7]. The Symlet and Coiflet wavelet come from Daubechie wavelet, but are more symmetric [8]. Both the scaling function and wavelet of Meyer are defined in frequency domain. Although the scaling function and wavelet of Meyer are symmetric, no fast algorithm is available for its wavelet transform [9]. The Morlet and Mexican wavelets only have wavelet functions and the corresponding scaling functions don't exist [10]. Four wavelets in three categories are selected for study here. They are orthogonal Daubechies wavelets and Butterworth wavelets with nonlinear phase, orthogonal Coiflets wavelets with near linear phase, and biorthogonal Spline wavelets with linear phase. All of them are associated with FIR filters except Butterworth wavelets.

Type	**Filter**	**Symmetry**	**Orthogonality**	**Fast algorithm**
Haar	FIR	Symmetric	Orthogonal	Yes
Daubechies	***FIR***	***Asymmetric***	***Orthogonal***	***Yes***
Symlets	FIR	Near symmetric	Orthogonal	Yes
Coiflets	***FIR***	***Near symmetric***	***Orthogonal***	***Yes***
Spline	***FIR***	***Symmetric***	***Biorthogonal***	***Yes***
Morlet	IIR	Symmetric	No	No
Mexican Hat	IIR	Symmetric	No	No
Meyer	IIR	Symmetric	Orthogonal	No
Butterworth	***IIR***	***Asymmetric***	***Orthogonal***	***Yes***

Table 1. Main properties of wavelet families

4. Comparison of Wavelet Bases

The comparison of $h_0(z)$ (low-pass filter for decomposition) for Db6, Coif4, Spline5.5 and Butterworth12 is illustrated in Figure 1. In order to show the difference between the phase linearity of Spline5.5 and Coif4, the relative errors of phase linearity are given in Figure 2. X. Chen used the most popular Daubechies wavelet to explore the potential application of wavelet transform in engineering surface analysis [1]. But Daubechies wavelet is not an ideal choice for application in engineering surface analysis because their amplitude transmission characteristics are not very sharp and phase responses are nonlinear. Butterworth wavelet has excellent transmission characteristics that lead to good frequency selectivity. But the disadvantage is the nonlinearity in the phase characteristics that are even worse than that of Daubechies wavelet families. For biorthogonal Spline5.5, the phase is linear. But the amplitude responses are rather poor. There are big variations in stopband and passband, and the gentle slope in transition band. The Coiflets has near linear phase. Although the amplitude response is not as good as that of Butterworth wavelet, it is much better than the transmission characteristics of Daubechies wavelet and Spline wavelet. Table 2 shows the results of the comparision.

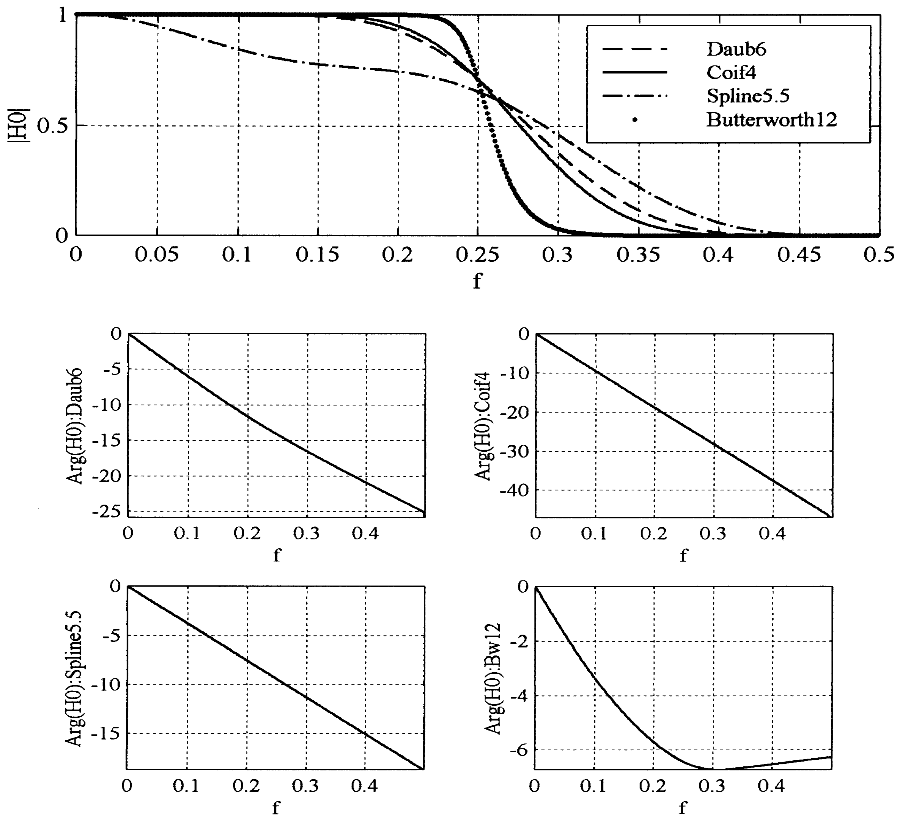

FIGURE 1. Comparison of Db6, Coif4, Spline5.5 and Butterworth12 wavelets

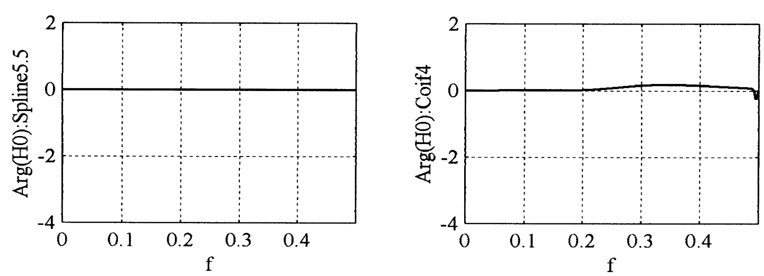

FIGURE 2. Relative phase errors of Spline5.5 and Coif4

	Db6	Coif4	Butterworth12	Spline5.5
Amplitude	not good	good	excellent	poor
Phase	nonlinear	near linear	nonlinear	linear

TABLE 2. Comparison of the wavelet features

5. Conclusion and Future Works

Wavelet analysis is a novel technique introduced into surface analysis in recent years. There are different wavelet bases for us to choose for analysis. This paper studied a set of wavelet bases in terms of the transmission characteristics associated with their corresponding scaling filter. There are other characteristics such as regularity and vanishing moments of the wavelets to be studies in terms of their impacts in surface analysis. Moreover, a set of surface analysis applications with wavelets needs to be done to validate the difference of applying different bases.

6. References

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