A New Optimal Design Technique for the Pneumatic Vibration Isolation System Implementing the Full Mathematical Analysis and Nonlinear Modeling

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1. Introduction

All precise machining and measurement equipments are much influenced by environments such as temperature, vibration, humidity, dust and so on. As for vibration, the amplitude of the vibration that resides in buildings and ground is from about 0.1 um to 10 um in general, which is bigger than the resolution of the precision equipments, and thus the vibration isolation systems are widely used in industry and laboratories.

The pneumatic vibration isolators are most widely used for the vibration isolation of small areas. However, their design depends on the empirical method and human experience. Many mechanical and mathematical analyses have been performed for the optimal design of the pneumatic vibration isolator, but the correspondence with real system is still weak.

The pneumatic vibration isolation system has nonlinear elements while the elements of the mass-spring-damper system are almost linear. For example, the orifice in the pneumatic system is a nonlinear element that the pressure drop between its both sides is proportional to the square of the flow rate. In previous researches, this element was regarded as only a linear element or as nonlinear one whose coefficients were assigned arbitrarily or artificially, so that the modeling might have less physical meaning. As a result, the full nonlinear characteristics of the pneumatic vibration isolation system has not been shown clearly.

In this research, a new analysis method is developed by modeling the nonlinear elements and calculating nonlinear algorithms numerically in order to show the full characteristics of the isolation system. These analyses and methods have achieved the optimal design of pneumatic vibration isolation system.

2. Description and modeling of the pneumatic vibration isolation system.

2.1 Composition

A typical vibration isolation system is shown in Fig.1, where it consist of two air chambers, an orifice between them, an elastomeric diaphragm which enables the sealing of air and the free movement of the payload, and a mass mounted on the piston.

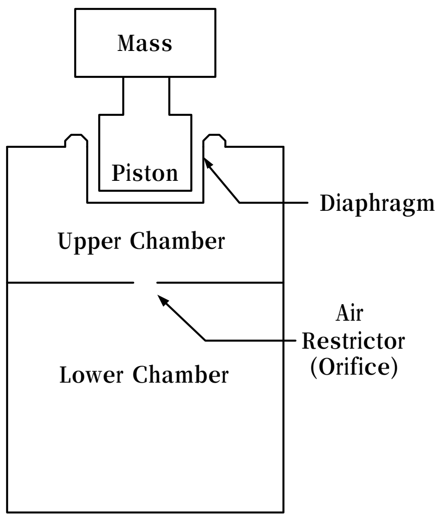

Fig. 1 Schematic diagram of a pneumatic vibration isolator

2.2 Air chambers

A sealed chamber with volume change behaves like a mechanical spring, where the relationship between pressure and volume in an air chamber can be described as follows.

$$P_0 V_0^{\,k} = PV^{k} \tag{1}$$

where k is the specific heat ratio of air, 1.4.

From this equation, we can derive the pressure change formula.

$$\Delta P = P - P_0 = P_0\left(\left(1 + \frac{\Delta V}{V_0}\right)^{-k} - 1\right) \tag{2}$$

2.3 Air restrictor (Orifice)

The flow rate through an sharp-edge orifice may be expressed as follows.

$$Q = \frac{C_d A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{\frac{2(P_1 - P_2)}{\boldsymbol{r}}} \tag{3}$$

where A_1, A_2: Pipe/Orifice Area [m^2]

P_1, P_2: Pressures in Pipe/Orifice [Pa]

C_d: Discharge Coefficient

In the case of the pneumatic vibration isolator, the pressure difference ratio of the upper and lower chamber does not exceed 10% and the ratio of the orifice diameter and the pipe diameter is very small. Thus we can set the discharge coefficient to be about 0.65.

And, equation (3) can be expressed like this by squaring both terms.

$$\Delta P_{ori} = -C_{ori}\left(\Delta \dot{V}\right)^2 sign(\Delta \dot{V}) \tag{4}$$

Because the pressure difference ΔP is analogous to the weight and the flow rate d(ΔV)/dt to the velocity, we can regard the orifice as a damper whose damping force is proportional to the square of velocity. This is the reason why the pneumatic system has nonlinear characteristics.

2.4 Diaphragm

The elastomeric diaphragm can be regarded as a damper and a spring. But, the stiffness of the diaphragm is constant in all loads while the stiffness of the air chamber changes in proportion to weight. Thus, the lighter is the mass, the more is the effect of the stiffness of diaphragm.

3. Simulation

3.1 Equations

As described above, the governing equations in the pneumatic vibration isolation system are as follows:

Relationship between the acceleration of the mass and the air pressure difference

$$\Delta P_1 = \frac{M}{A_p}\ddot{x}_{piston} \tag{5}$$

Pressures in the pneumatic vibration isolator

$$\Delta P_1 = \Delta P_2 + \Delta P_{ori} \tag{6}$$

where

$$\Delta P_1 = P_0\left(\left(1+\frac{\Delta V_1-\Delta V_2}{V_1}\right)^{-k}-1\right) \tag{7}$$

$$\Delta P_1 = P_0\left(\left(1+\frac{\Delta V_2}{V_2}\right)^{-k}-1\right) \tag{8}$$

$$\Delta P_{ori} = -C_{ori}\left(\Delta \dot{V}_2\right)^2 sign(\Delta \dot{V}_2) \tag{9}$$

Relationship between the ground excitation and the upper chamber volume change

$$\Delta V_1 = A_p(x_{piston} - x_{ground}) \tag{10}$$

3.2 Algorithm

Using the relationship equations in section 3.1, a simulation has been carried out in the time domain by MATLAB/Simulink. Followings are the steps in the program:

- Ground excitation invokes the volume change of the upper air chamber and its pressure change.
- The pressure change makes the air flow through the air restrictor.
- The flow changes the volume of upper and lower air chambers and their pressure changes.
- The change of the pressure difference between two chambers results in the change of the air flow through the orifice
- The pressure change of the upper air chamber makes the change of the supporting force under the piston.

These steps have been accomplished iteratively by numerical analysis and Runge-Kutta method was used to solve differential equations.

3.3 The calculation of the transmissibility

The result data of ground excitation and table-top vibration by the time-domain simulation goes through FFT algorithm because the transmissibility can be attained in the frequency domain. The transmissibility versus the excitation amplitude and frequency is shown in Fig. 2.

If the elements of the pneumatic system are assumed as linear ones, the transmissibility has nothing to do with the amplitude, because the amplitude coefficients in the linear differential equations can be vanished by division. The transmissibility, however, has been found out to be dependent on the amplitude.

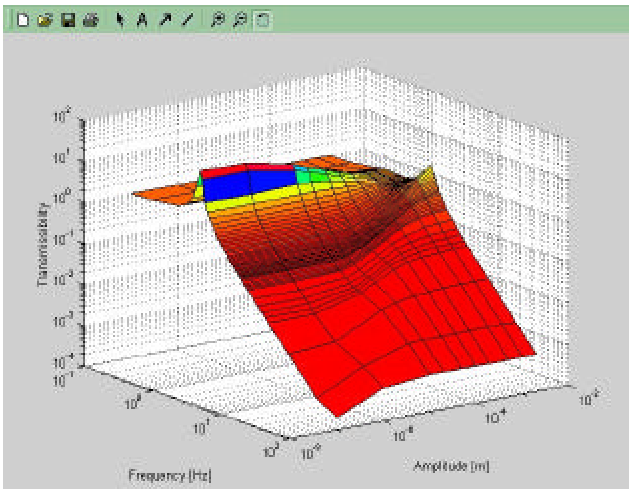

Fig. 2 Transmissibility of a vibration isolator

4. Comparison

4.1 Trend

As seen in the transmissibility graph, when the orifice area is small or the excitation amplitude is large or the excitation frequency is high, the effect of the upper air chamber alone is dominant. In the opposite case, however, the net volume of the both air chambers comes to be in effect. It shows good accordance with the description of one pneumatic vibration isolator maker catalog.

4.2 Natural frequency

We simulated the pneumatic system with following conditions.

- Effective piston area: 1.742e-3 m^2
- Mass: 61.3 Kg
- Upper chamber volume: 71.44e-6 m^3
- Lower chamber volume: 507.6e-6 m^3
- Orifice area: 384.8e-9 m^2

From the simulation, the resonant frequencies are 1.0 Hz in the small excitation amplitudes and 2.9 Hz in the large excitation amplitude respectively. For the verification of the simulation, a prototype has been made with the same specification. In the large amplitude of excitation, the resonant frequency of the system was measured as about 3.0 Hz, producing very close value of simulation. Thus it can be stated that the simulation can be a good tool to optimize the vibration isolation system.

5. Optimal design

Because all design parameters such as chamber volume, operation air pressure, and so on, can be input to the simulation program, the influence of the each design parameter can be known.
Followings are the representative design parameters to be optimized:

- Chamber volumes
- Piston Area
- Orifice Area

6. Conclusion

The proposed modeling and nonlinear simulation made clear the effect of each components of the pneumatic vibration isolation system enabling the optimal design based on the good agreement between experiments and simulation. When the design parameters are input to the simulation program, their effects can be observed successfully. Therefore, the design parameters can be optimally chosen by the simulation results.

References

1. K. Peter Heiland, "Recent Advancements in Passive and Active Vibration Control Systems", SPIE Vol.1619, Vibration Control in Microelectronics, Optics and Metrology, 22-33, 1991
2. G. Popov and S. Sankar, "Modelling and Analysis of Non-linear Orifice Type Damping in Vibration Isolators", J. Sound and Vibration, 183(5), 751-764, 1995
3. E. O. Doebelin, *Measurement Systems : Application and Design,* 4th Ed., McGraw-Hill, p.566, 1990
4. Kinetic Systems Inc., *Vibration Isolation Products Catalog,* 1999