ABSOLUTE STRAIN MEASUREMENT FOR FIBER BRAGG GRATING SENSOR USING A STRING RESONATOR WITH HIGH ACCURACY

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ABSTRACT

This paper describes a string resonator that is used for the interrogation system of a Fiber Bragg Grating (FBG) strain sensor. The strain on the fiber piece is calculated from the measured frequency based on that the natural frequency of a string is a function of the applied absolute strain. Existing research considered a fiber as a string, but a fiber is not a string in the strict sense due to its bending stiffness, thus the fiber should be modeled as a beam accompanied with an axial force. In the vibration modeling, the relationship between the strain and the natural frequency is derived, and then the resonance condition is described in terms of both the phase and the mode shape for sustaining resonant motion. Several experiments verify the effectiveness of the proposed model of the fiber.

Keywords: absolute strain, string resonator, fiber Bragg grating sensor, tunable-filter, high accuracy, resonance, beam model, string model

1. INTRODUCTION

Fiber Bragg Grating (FBG) is an optical fiber element, which is fabricated to show periodic change of the refractive index of the fiber core and reflects a narrow band of wavelength, called Bragg wavelength, when illuminated with a broadband light source. The principle of a FBG sensor is based on the fact that the Bragg wavelength of the sensing element shifts when strain or temperature change arises in the element. FBG sensors provide advantages such as distributed sensing using only single fiber line, ease of insertion and attachment, self-referencing capability independent of the total power level of the source and the loss of coupler or connection, high sensitivity, electromagnetic interference immunity, and multiplexing capability [1][2].

With wavelength interrogating method using Fiber Fabry-Perot (FFP) filter, a real-time strain measurement was achieved for wavelength division Multiplexing (WDM) with four FBG's, and quasi-static resolution of ~0.3με and ±3με was obtained for single and four FBG's, respectively [3]. However, this method shows inaccuracy in wavelength interrogation due to the hysteresis and the non-linearity of piezo actuator. To overcome this problem, a new interrogation system consisting of a FBG as the filter replacing FFP and a string oscillator for an absolute strain measurement was suggested[4]. By introducing the string oscillator that automatically sustains the fiber motion in phase with the excitation, an accuracy of ±2με and a quasi-static resolution of ~0.05με in 1200με dynamic range were reported. Despite the improvement, there exists a problem in this new system that the measured absolute strain was based on a modeling of the fiber as an ideal string, which could affect the validity of the measured strain.

In this work, a robust modeling of the string resonator is conducted, considering the stiffness of the string. A string resonator is designed and implemented to evaluate the effectiveness of the proposed model and the actual performance of the strain measurement system.

2. VIBRATION ANALYSIS

2.1 Natural frequency and strain.

Neglecting the damping of an ideal string, the motion equation for free vibration and the corresponding natural frequency are given by

$$P\frac{\partial^2 w(x,t)}{\partial x^2} = \rho_L \frac{\partial^2 w(x,t)}{\partial t^2} \qquad (1)$$

$$f_i = \frac{i}{2L}\sqrt{\frac{P}{\rho A}} \qquad (2)$$

where P is tension, L is length of the string, ρ and ρ_L are the density and linear density of the string material respectively, and

f_i is the i-th mode natural frequency in Hz. Eqn.(2) can be rewritten in terms of the strain as follows,

$$f_i = \frac{i}{2L_0(1+\varepsilon)}\sqrt{\frac{E\varepsilon}{\rho}} = \left(\frac{i}{2L_0}\sqrt{\frac{E}{\rho}}\right)\frac{\sqrt{\varepsilon}}{(1+\varepsilon)} \tag{3}$$

where L_0 is length of the string without strain. Under the assumption of small strain, the natural frequency varies linearly to the square root of the strain. Therefore, it is possible to calculate the strain from the measured natural frequency [4].

In case of a fiber, however, it is difficult to neglect the bending stiffness because the fiber cannot be assumed as an ideal string any more. Instead, the fiber should be considered as a beam, and then the equation of vibration is rewritten as

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} - P\frac{\partial^2 w(x,t)}{\partial x^2} = 0 \tag{4}$$

The solution of Eqn.(4) can be expressed as $w(x,t) = W(x)(A\cos\omega t + B\sin\omega t)$. Using an assumed mode method, the general solution of Eqn.(4) is represented by

$$w(x,t) = \sum_{i=1}^{\infty}\Phi_i(x)q_i(t) \tag{5}$$

$$\int_0^L \rho A\Phi_i(x)\Phi_j(x)dx = \delta_{ij} \tag{6}$$

where, Φ_{ij} is mode shape of the vibration, and δ_{ij} is Kronecker's delta. Eigenvalue problem is drived from Eqn. (4) as follows [5].

$$[I]\{\ddot{q}\} + [K]\{q\} = \{0\} \tag{7}$$

$$K_{ij} = \omega_{b,i}{}^2\delta_{ij} + P\int_0^L \Phi_i{}'\Phi_j{}'dx = \omega_{b,i}{}^2\delta_{ij} + P\Gamma_{ij} \tag{8}$$

$$q(t) = A\cos\omega_i t + B\sin\omega_i t \tag{9}$$

In above equation, $\omega_{b,i}$ is the natural frequency of the beam without axial tension and matrix Γ_{ij} is defined by Eqn. (10), (11).

$$\Gamma_{ij} = \int_0^L \Phi_i^{'}\Phi_j^{'}dx \quad (i \neq j) \tag{10}$$

$$\Gamma_{ij} = \int_0^L \left(\Phi_i^{'}\right)^2 dx \quad (i = j) \tag{11}$$

By applying the boundary conditions for clamped beam, the characteristic equation of the beam with an axial force is derived as

$$\left|P\Gamma_{ij} - \left(\Lambda - \omega_{b,i}{}^2\right)\delta_{ij}\right| = 0 \tag{12}$$

where, Λ is the eigenvalue and equal to the square of the natural frequency $\omega_{b,j}$ (rad/sec) of the beam with tension. Considering that the length and cross-sectional area of the beam vary with the strain change, Eqn.(12) becomes,

$$\left|\left\{\frac{\varepsilon}{(1+\varepsilon)^2}\right\}EA_o\Gamma_{o,ij} - \left\{\Lambda - \frac{\omega_{bo,i}{}^2}{(1+\varepsilon)^3}\right\}\delta_{ij}\right| = 0 \tag{13}$$

The solution of Eqn.(13) obtained using MATLAB is compared to that of Eqn.(3)

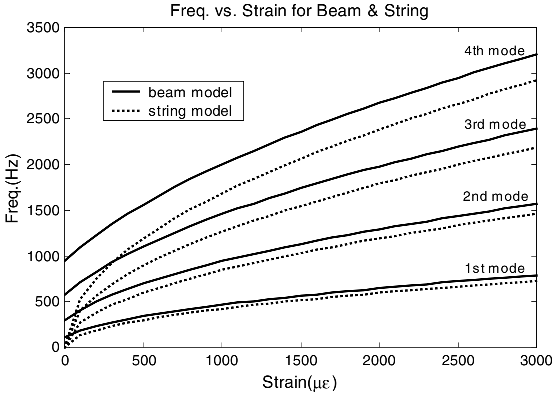

Fig.1 Natural frequency vs. strain
(solid line: beam model, dotted line: string model)

The main difference between the beam and string model is that the natural frequency of the beam model has nonzero values while that of the string model is always zero when no strains exist on the fiber.

2.2 Resonance Condition

When a fiber is constrained at both ends while one of the them is excited according to Eqn.(14), the relative motion of the fiber can be represented by Eqn.(15) [6],

$$w_g = \delta(x)g(t) \tag{14}$$

$$w^* = -\sum_{i=1}^{4}\frac{\rho A\Phi_i(x)}{\omega_i}\left[\int_0^L \delta(x)\Phi_i(x)dx\int_0^t \ddot{g}(t^{'})\sin\omega_i(t-t^{'})dt^{'}\right] \tag{15}$$

where $\delta(x)$ is a displacement influence function. Thus, the total displacement of the fiber is represented as the sum of a rigid-body motion, Eqn.(14), and the relative motion, Eqn.(15), as follows

$$w = w_g + w^* = \delta(x)g(t) + w^* \tag{16}$$

Assuming that the excitation can be represented by a sinusoidal function as $g(t) \equiv A_g \sin\Omega t$, the total motion in Eqn.(16) can be rewritten as

$$w = A_g \delta(x)\sin\Omega t + \sum_{i=1}^{4} \rho A \Phi_i(x) \left[\int_0^L \delta(x)\Phi_i(x)dx \right] A_g H(\Omega, t) \tag{17}$$

$$H(\omega_k, t) \approx \frac{\omega_k t}{2} \sin\left(\omega_k t - \frac{\pi}{2} \right) \tag{18}$$

From Eqn.(18), it is shown that the fiber motion is delayed as much as π/2 than the excitation motion for 1st mode. And for 3rd mode, fiber motion is faster as much as π/2 than the excitation motion..

3. EXPERIMENTS AND RESULTS

3.1 Experimental setup

The functional diagram of automatic string resonator is shown in Fig.2. The flexure stage of the string resonator is composed of two elastic hinges of notch type [7]. One flexure is for fiber stretching with a piezo actuator, PZT1, and another is for excitation motion, PZT2.

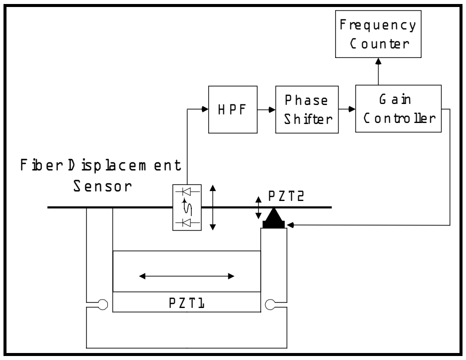

Fig. 2 Functional diagram of string resonator

A photo interrupter, which is composed of a pair of a LED and a phototransistor, was used as the displacement sensor measuring the fiber's oscillating motion. The signal from the photo interrupter, which represents the displacement of fiber center, goes through high-pass filter so that DC offset is removed. The phase of the AC signal from the high-pass filter shifts by 90° by the integrator for resonant condition. The automatic gain controller makes the amplitude of the output signal be constant regardless of frequency.

3.2 Results

Using the string resonator circuit the resonant frequencies with respect to the strain are measured. The relative strain to the initial value is calculated from the displacements read by a capacitance sensor, and the resonant frequency is read from a frequency to voltage converter [8]. The experiments are mainly to detect the 1st mode and 3rd mode resonance frequency.

Fig.3 and Fig.4 shows measured natural frequency with respect to strain, and error from fitting equation, Eqn.(13) at 1st and 3rd modes.

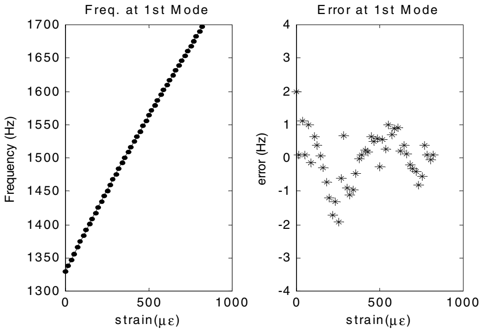

Fig. 3 Fitting of the experimental data at 1st mode

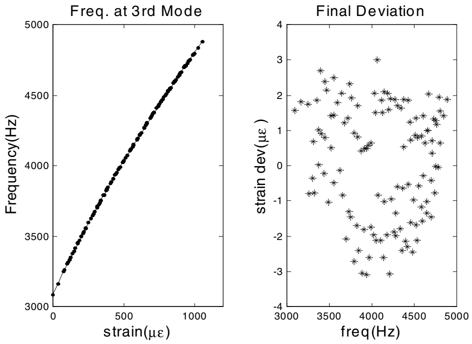

Fig. 4 Fitting of the experimental data at 3rd mode

To test resolution, the frequency variation with respect to a small step strain (0.7με) is measured in each mode, Fig.5 and Fig.6. For 1st and 3rd mode, it shows accuracy of 4με and 3με and quasi-static resolution of 0.2με(rms) and 0.1με(rms), respectively.

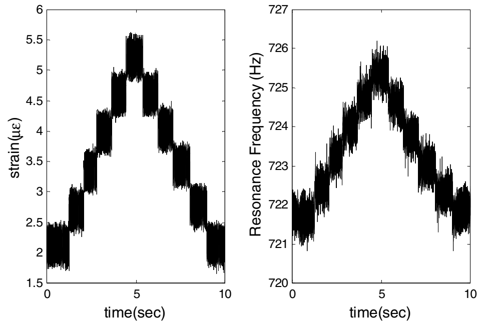

Fig. 5 Response to small step strain at 1st mode

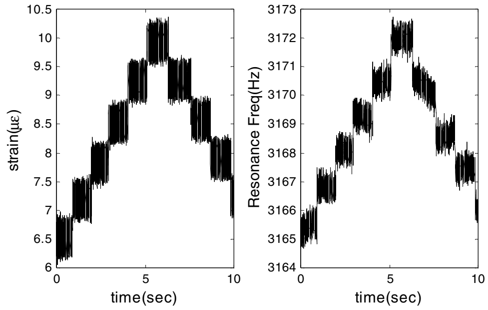

Fig. 6 Response to small step strain at 3rd mode

Frequency response of the 3rd mode with respect to a dynamic strain is measured for input signal with the amplitude of 25με. The frequencies of the strain input were changed from 1Hz to 16 Hz, Fig.7. For frequencies smaller than 8Hz, excellent locking capability is achieved, with an accuracy of ~3με and almost no phase difference.

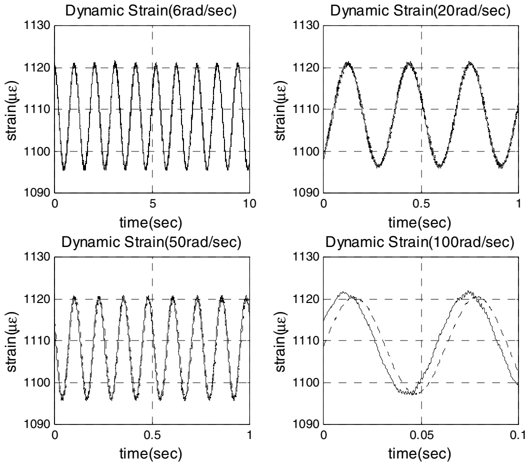

Fig. 7 Response to dynamic strain

4. CONCLUSION

By the vibration analysis, the measuring scheme for the absolute strain was established, from which the string resonator for the locking filter interrogation system was implemented. The measurement system of 3rd mode detection provides the sensitivity of three times compare with 1st mode, the accuracy of ±3με, and the quasi-static resolution of ~0.1με which are better than 1st mode. For dynamic strain it provides accuracy of ~3με below than 8Hz input. Consequently, the performance of the string resonator for the absolute strain measurement with high sensitivity and high accuracy can be advanced by the design improvement of the flexure and the novel scheme for resonance detection of 3rd mode.

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