Measurements using Fourier Transform Phase Shifting Interferometry

Leslie L. Deck
Zygo Corporation, Laurel Brook Road, Middlefield, CT. 06455-0448

Phase shifting interferometry[1] (PSI) using wavelength tuning is a preferred technique for high-precision surface form measurements of large optics[2]. Recently, a new phase shifting interferometry technique was described that has a significantly greater flexibility and measurement precision than traditional PSI[3]. Called Fourier transform phase-shifting interferometry (FTPSI)[4], the technique combines phase shifting via wavelength tuning with specific cavity geometries and a flexible Fourier based analysis technique to identify and extract surface features from many surfaces simultaneously. In addition, FTPSI preserves the spatial relationships between the physically separated surfaces so that measurements of surface profiles, physical wedge and homogeneity can be easily extracted. These measurements have been demonstrated for a parallel plate in Ref. 3. In this paper, I describe these and several additional measurement possibilities using FTPSI.

1. Fourier transform phase-shifting interferometry

The technique is based on the realization that to first order, a multi-surface cavity is just a combinatorial collection of elemental (2-surface) cavities, and that with wavelength tuning, each elemental cavity generates interference with a phase variation directly related to the cavities' total optical path difference (OPD) nT and the optical frequency tuning rate $\dot{\nu}$ via

$$\dot{\varphi} = 2\pi nT\dot{\nu}/c\,, \tag{1}$$

where n is the index and T is the cavity physical thickness. If the OPD of each elemental cavity is unique, each cavity then produces interference at a frequency f_C given by,

$$f_C = nT\dot{\nu}/c\,. \tag{2}$$

A Fourier analysis of the interference variation from a single wavelength-tuned measurement extracts the spectrum of interference frequencies. Each spectral peak corresponding to the interference from an elemental cavity of interest is individually analyzed for the interferometric phase φ using a windowed discrete Fourier transform evaluated at the frequency of the peak;

$$\varphi = \tan^{-1}\left(\frac{\mathrm{Im}(\mathrm{DFT}(f_C))}{\mathrm{Re}(\mathrm{DFT}(f_C))}\right), \tag{3}$$

with

$$\mathrm{DFT}(f_C) = \sum_{j=0}^{N-1} I_j W_j \exp\left(i2\pi j f_C / f_S\right). \tag{4}$$

The I_j represent the N intensity samples acquired during the wavelength tune, W_j are the sampling weights and f_S is the sampling rate. The optical frequency variation $\dot{\nu}$ is assumed linear throughout the acquisition. As with PSI, surface profiles are obtained from the spatial variation of the interferometric phase. Figure 1 shows the apparatus used to demonstrate the technique, incorporating a conventional Fizeau interferometer outfitted with a wavelength tunable laser.

To minimize the effect of higher order interference frequencies disturbing the phase evaluation, cavity geometries are used which separate all the 1st and 2nd order frequencies from each other. This entails selecting the total optical frequency excursion, the primary gaps L_1 and L_2 and the number of intensity samples N, depending on the value of the test optic OPD, nT. The details of this selection procedure are discussed in Ref. 3, the main result being that the optimal cavity geometry is one in which the ratio of the optical path lengths of any two primary gaps is a unique power of three.

2. Relative Measurements

The 4-surface cavity shown in Figure 1 can be used to measure many properties of a test optic that is bounded by two parallel surfaces – a parallel plate being the canonical example. The four surfaces making up the cavity decompose into six elemental 2-surface cavities, each producing a unique 1st order interference frequency. The interferogram recorded at each pixel is spectrally decomposed with a Fourier transform and the interference frequency peaks corresponding to the elemental cavities are identified. Figure 2 shows the interference frequency spectrum obtained when using the 4-surface geometry. The spectrum of 2nd order cavity frequencies is also shown to highlight the excellent separation between the 1st and 2nd order peaks.

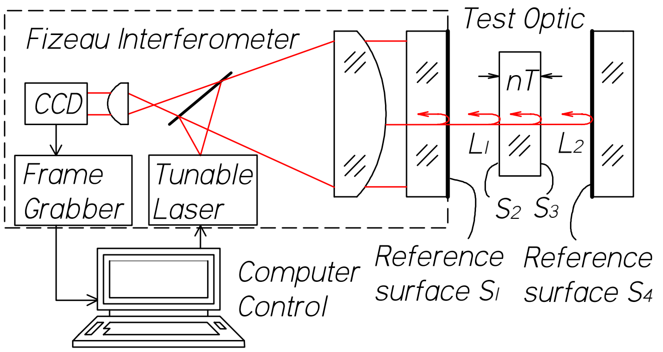

Figure 1: Apparatus used to demonstrate the method.

Each 1st order frequency peak corresponds to a particular elemental cavity. The spatial phase variation of each 1st order peak evaluated at the 1st order frequency represents the OPD variation between those two surfaces. For example, assuming $nT < L_1 < L_2$, the first leftmost peak corresponds to interference between the two surfaces of the test optic S_2 and S_3. As such, the spatial phase map can be used to measure the optical thickness variation. The second peak from the left corresponds to interference between the first reference surface S_1 and the first surface of the test optic S_2. Its phase variation corresponds to the first surface profile of test optic. Similarly, the profile of the second surface of the test optic S_3 relative to the second reference S_4 is found from the fourth peak. If necessary, S_3 can be referenced to S_1 with an additional measurement of the

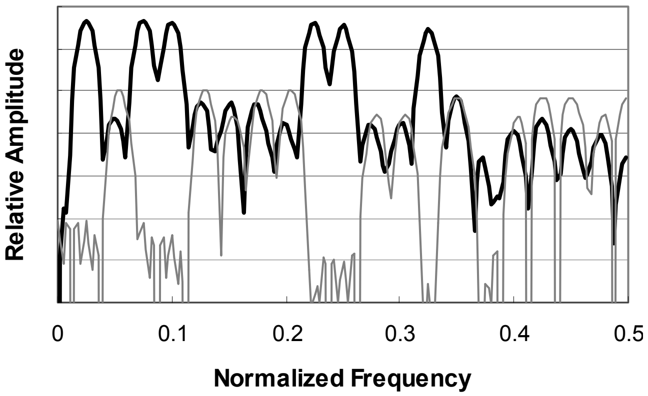

Figure 2: Log plot of the interference spectrum for the optimum 4-surface geometry normalized to the sampling rate. The heavy curve is the full spectrum and the light curve is the spectrum of 2nd order frequencies.

cavity with the test optic removed.

The information for each elemental cavity is obtained with a single wavelength tuned acquisition. The simultaneous nature of these measurements implies that spatial relationships between the surfaces are preserved. This allows the measurement of index homogeneity from the difference between the optical thickness variation and the physical thickness variation obtained from the empty cavity measurement and the measurements of the two outermost primary elemental cavities, S_1:S_2 and S_2:S_3. Surface profiles, physical and optical wedge and relative homogeneity measurements were demonstrated in Ref. 3, this paper will concentrate on additional measurement capabilities of the technique.

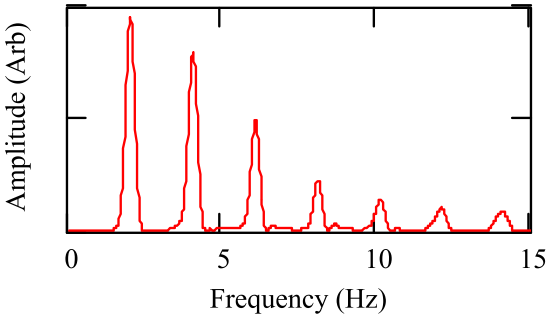

Figure 3: Plot of the interference spectrum from a high finesse cavity.

3. High finesse cavities

When the reflectivities of the surfaces making up an elemental cavity are high, the optical power transferred to higher order multiple interference events increases, producing interference at harmonics of the 1st order frequency. The distorted fringe pattern produced cannot be analyzed with conventional PSI without significant error. FTPSI separates these harmonics so that the 1st order frequency can be analyzed in isolation, producing distortion free profiles of the surfaces. Figure 3 shows the frequency spectrum obtained from a high finesse cavity whose interference pattern is shown in Fig. 4

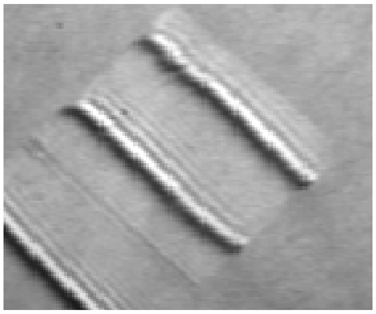

Figure 4: High finesse fringes.

4. Internal angle measurements

By selecting which elemental cavity to analyze, measurements of internal angles of various optical elements can also be realized. Figure 5 shows one possible geometry for measuring the 90^O angle and the pyramidal error of a right angle prism. To 1st order, a ray encounters 4 surfaces before leaving the cavity, S1, S2, S2` and S1`. The S1:S2 cavity provides a measure of the S2 surface, which acts as the orientation referent for the non-retro direction for the pyramidal error. The tilt of the S1:S1` cavity in the retro direction determines the 90^O angle. Note that the direct return of the beam back to the camera forces the use of a camera with good dynamic range. The dashed line to the left of S1 indicates where a beam stop can be used to remove the direct return if necessary. A traditional PSI measurement of this angle requires the removal the S1 surface and measures the S2:S2` cavity, however the geometry shown here allows us to simultaneously qualify the S2 surface against a known reference.

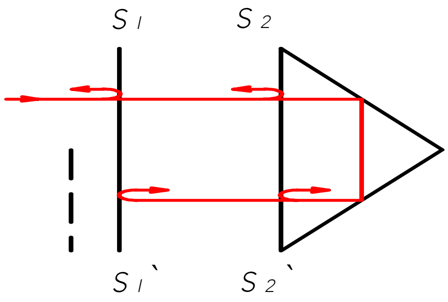

Figure 5: Cavity geometry for measuring the 90^O angle and pyramidal error of a right angle prism.

5. Absolute measurements

Equation 1 implies that the total cavity OPD can be obtained if the total phase shift is measured during a known optical frequency excursion. This process is known as coherent ranging[5]. To account for the spectral dispersion of the test optic, Eq.1 must be modified to

$$\dot{\varphi} = \frac{2\pi n T \dot{v}}{c}\left(1 + \alpha v / n\right) \tag{5}$$

where α is the 1st order dispersion coefficient of the test optic material. The magnitude of this correction is of order 1% for typical glasses at wavelengths from the visible through the near-IR. The test optic absolute physical thickness T can be found with the 4-surface cavity geometry via

$$T = \frac{c}{2\pi \dot{v}}\left(\dot{\varphi}_{14} - \dot{\varphi}_{34} - \dot{\varphi}_{12}\right). \tag{6}$$

Here $\dot{\varphi}_{14}$ is the phase variation observed in the empty cavity and the 1st order dispersion coefficient for air is assumed to be zero. The absolute index of the test object is obtained by solving Eq. (5) for nT and dividing by Eq. (6)

$$n = \frac{\dot{\varphi}_{23}}{\left(\dot{\varphi}_{14} - \dot{\varphi}_{34} - \dot{\varphi}_{12}\right)} - \alpha \frac{c}{\overline{\lambda}}, \tag{7}$$

where $\overline{\lambda}$ is the mean value of the wavelength used. Note that for the absolute index, the actual wavelength used must be known, while for the thickness only the change in the wavelength during the measurement is required.

6. Summary

I have outlined a new general methodology for optical profiling using wavelength tuning combined with specific cavity geometries and an effective Fourier based analysis technique. Called Fourier transform phase-shifting interferometry (FTPSI), the method is a general replacement for traditional PSI. This paper has outlined some of the more interesting measurement possibilities of the technique, including;

1) Surface profiles from multiple surface cavities
2) Surface profiles from high finesse cavities
3) Internal angle measurements
4) Measurements of relative and absolute optical thickness
5) Measurements of relative and absolute physical thickness
6) Measurements of relative and absolute index homogeneity

7. References

---

[1] J. E. Greivenkamp and J. H. Bruning, "Phase Shifting Interferometry," *Optical Shop Testing*, D. Malacara, 501-598, J. Wiley, New York, 1992

[2] L. Deck, J. A. Soobitsky; "Phase-shifting via wavelength tuning in very large aperture interferometers," Proc. SPIE, 3782-58, 432-442 (1999)

[3] L. Deck, "Multiple surface phase-shifting interferometry," to be published in Proc. SPIE, 46th Annual, San Diego, (2001)

[4] FTPSI is the topic of US and Foreign patents pending assigned to Zygo Corporation

[5] P. de Groot, "Chromatic dispersion effects in coherent absolute ranging," Opt. Lett., **17**, 898-900 (1992)