Static modeling of grinding wheel

Hwa-Soo LEE, Kojiro SADASUE, Yujiro ITO and Tadashi WAKABAYASHI

Department of Mechanical Engineering, College of Science & Technology,
Nihon University.
1-8-14 Kanda-Surugadai, Chiyoda-ku, Tokyo, 101-8308, Japan

Abstract

In this study, in order to clarify dynamic phenomena of grains in the contact area with workpiece, a dynamic model of wheel is proposed and an experimental evaluation of this model for applications to dynamic analysis is carried out. As the first step of this study, a static wheel model is proposed in which rigid spherical grain particles are suspended by simple bonds having static stiffness k. In this study, elastic deformation of wheel is evaluated by FEM and by three points bending test. As the results of these considerations, it is confirmed that the wheel model proposed in this study can be used to the static analysis on elastic deformations in contact area with workpiece.

Key words : Grinding wheel, Contact stiffness, Static Stiffness, Static analysis.

1. Introduction

Static and dynamic phenomena in grinding operations are affected by dynamics between grinding wheel and workpiece consisting of workpiece, grinding wheel and contact stiffnesses. In these parameters, contact stiffness between wheel and workpiece has been analyzed by applying the theory of Hertzian contact so far/1, 2, 3/. In these analysis, elastic deformations in contact area are discussed by using elastic modulus and Poisson' ratio of grinding wheel under the assumption that wheel is uniform continuous body. As well known, however, wheel consisting of abrasive grain, bond and void is not actually uniform continuous body. Furthermore, the number of abrasive grains in small contact area in grinding operation is finite. Consequently, it may be some problems that wheel is assumed to be a uniform continuous body in order to analyze the dynamic phenomena in contact area.

This study aims to propose a kinetic model of wheel used to analyze the phenomena in contact area between wheel and workpiece and to prove the propriety of the proposed model analytically and experimentally. As the first step of this study, a static model consisting of rigid spherical abrasive grains and bonds connecting successive two grains which has static stiffness k is proposed. In this study, in order to clarify the propriety of the proposed kinetic model, theoretical analysis by finite element method and experimental analysis by three point bending tests are carried out.

2. Analytical approach

2.1 Kinetic model of wheel

Figure 1 shows an image of three-dimensional uniform arrangement of idealized sphere grain in a grinding wheel and/or abrasive stick. A kinetic model in which bond is connecting successive two bonds as shown in Figure 2 is proposed. Assuming that these sphere grains are arranged uniformly in grinding wheel and/or abrasive stick, amount of static stiffness k connecting two grains can be calculated by the following a method.

Assuming that the model shown in Figure 2 is a unit of grain arrangement, volume percentage of grain in wheel Vg can be shown as the function of total number of grains N as:

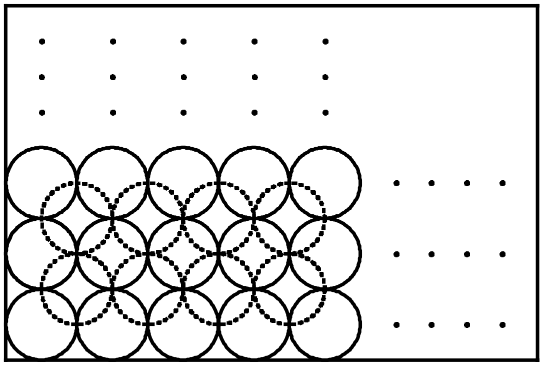


Figure 1 Ideal arrangement of sphere abrasive grains

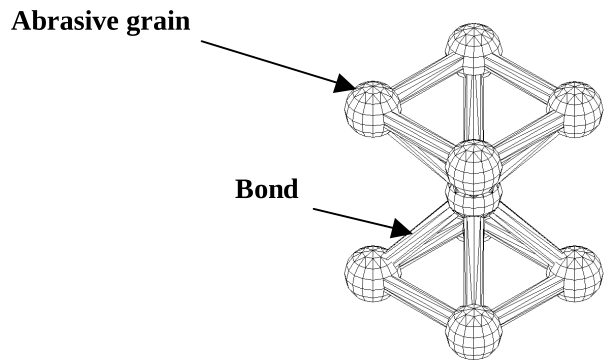


Figure 2 Unit of kinetic model of grain arrangement

$$V_g = \frac{\left(4/3\right)\pi\, r^3 \times N}{V} \qquad (1)$$

Where V is a volume of whole wheel. Distance between successive two grains can also be calculated geometrically as a function of volume percentage of grain in wheel.

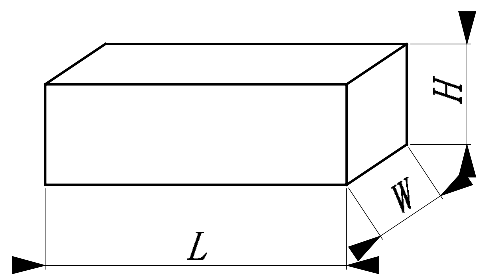
2.2 Calculation of static stiffness k

Supporting an abrasive stick shown in Figure 3 in two points, displacement of the center where load P is applying is shown as follows.

$$\delta = \frac{PL^3}{48E'I'} \qquad (2)$$

Where L is the length of stick, E' and I' are equivalent elastic modulus and moment of inertia of area respectively. On the other hand, analyzing the relation between load P and the displacement at loading point δ in three points bending test by finite element method, these relation can be shown as a function of P and k.

Grinding stick	WA60 J V	WA120 J V
Grain size	＃60 (=0.25[mm])	＃120 (=0.105[mm])
Grain volume percentage	50 [%]	50 [%]



Stick	**Width [mm]**	**Height [mm]**	**Length [mm]**
1	8.0	5.7	40
2	4.0	5.7	40
3	5.0	7.1	40
4	7.5	6.9	40
5	5.7	8.0	40
6	5.7	4.0	40

Figure 3 Abrasive sticks used in experiment

$$\delta = \alpha \frac{P}{k} \qquad (3)$$

Where, α is a coefficient obtained in FEM, which is determined by the number of springs and the arrangement of abrasive grains. From the relation shown in Equations (2) and (3), static stiffness of each bond k can be obtained as a function of E' and I'.

$$k = \frac{48E'I'}{L^3} \cdot \alpha \qquad (4)$$

2.3 Analytical results

In the method shown above, the coefficients α are calculated and shown in Figure 4. According to the calculated results, it is known that α has a tendency to decrease with the increase of volume percentage of grains in wheel.

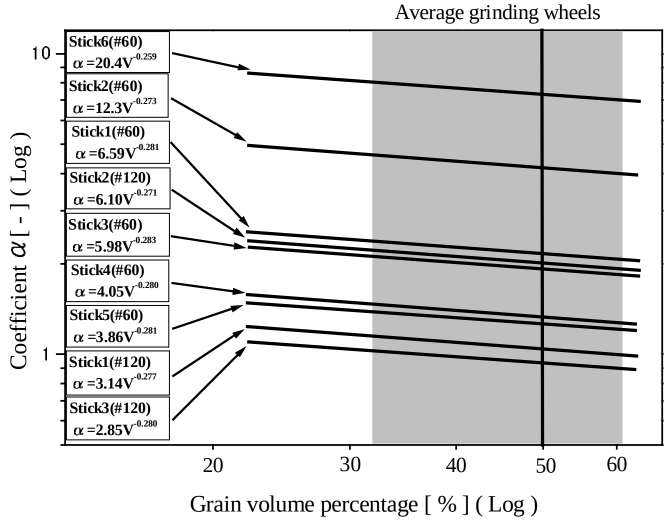


Figure 4 Analyzed results

3. Experiments and results

Experimental results of three points bending test for WA60JV and WA120JV are shown in Figure 5 and 6 respectively. From these figures showing the relations between relative load and relative displacement, it can be known that their relations depend on the geometrical shapes and sizes of abrasive stick. Equivalent elastic modulus E' obtained from the inclinations of the test results shown in Figure 5 and 6 are shown in Table 1. In this table, static stiffness k and equivalent moment of inertia of area I' obtained form the theoretical analysis described above are also shown. These analytical results show that equivalent elastic modulus E' and equivalent moment of inertia I' vary depending on the geometrical shape of tested abrasive stick. On the other hand, the static stiffness k are constant independently from the geometrical shape and are almost 8.00[N/μm] for #60 stick and 4.00[N/μm] for #120 stick respectively. This tendency means that the equivalent elastic modulus of the whole abrasive stick, which has been assumed to be constant, changes depending on the geometrical shape, but the static stiffness k of the basic unit of abrasive stick is independent from the shape. Consequently, using the kinetic model proposed in this study, elastic phenomena of grinding wheel may be analyzed independently from geometrical shape of wheel and/or stick.

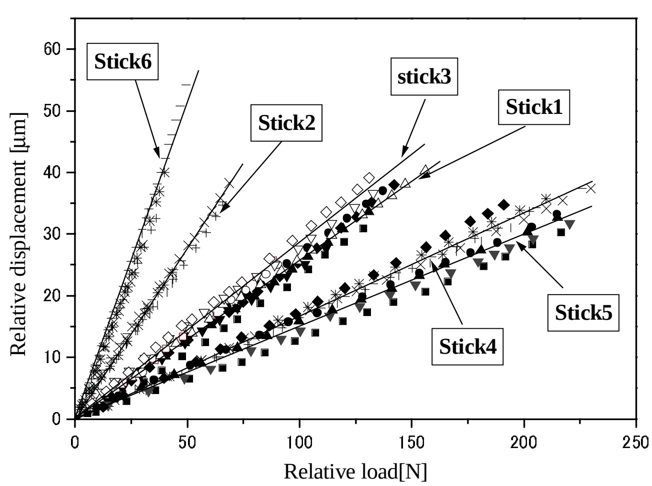


Figure 5 Results of three points bending tests (#60)

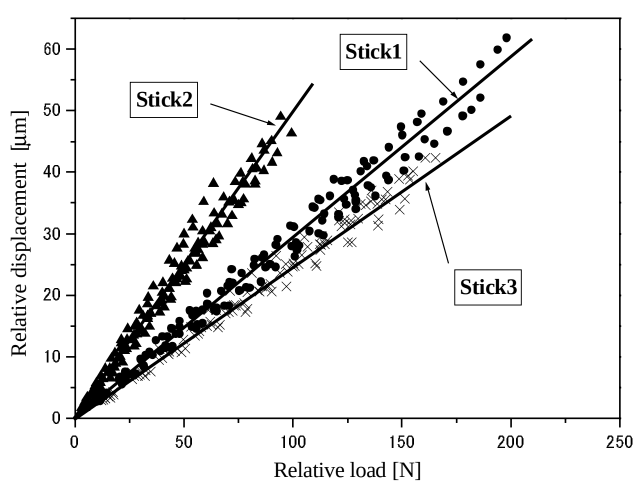


Figure 6 Results of three points bending tests (#120)

Table 1 Results

	Stick	Equivalent elastic modulus E[kN/ mm^2]	Static stiffness k[N/ μ m]	Equivalent moment of inertia of area I [mm^4]
#60	**1**	41.5	**8.42**	123
	2	40.7	**7.97**	61.7
	3	32.5	**7.24**	149
	4	39.9	**8.29**	205
	5	36.9	**8.61**	243
	6	43.3	**7.32**	30.4
#120	**1**	37.3	**3.67**	123
	2	44.3	**4.33**	61.7
	3	35.7	**3.80**	149

According to the test results, it is known that static stiffness in #60 stick is two times larger than the stiffness in #120 stick. From such tests results, it may be considered that the amount of static stiffness changes by the size effect of bond connecting successive two abrasive grains as follows. That is, since the size of bond may be considered to be changed with the grain size, the length of bond in #60 stick is two times longer than the length of bond in #120 stick and the sectional area of bond in #60 stick is four times larger than one in #120 stick as shown in Figure 7. It means that the amount of static stiffness k shown in Figure 2 is affected by the bond size and then the amount of k in #60 stick may be two times larger that one in #120 stick. From such a consideration, the test results shown above may be understood easily.

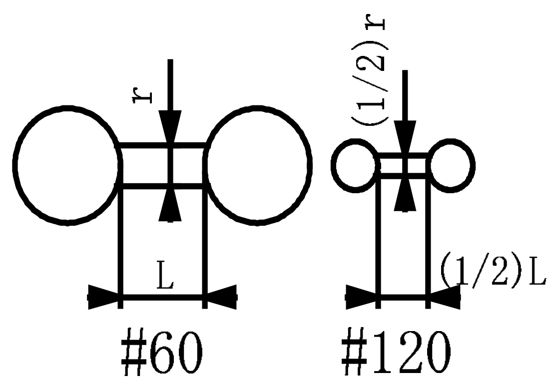


Figure 7 Bond sizes

4. Conclusions

(1) A static model for analyzing the elastic phenomena of wheel is proposed.

(2) As the results of theoretical and experimental analysis, it is clarified that the model proposed in this study can be used to analyze the elastic phenomena of wheel.

Acknowledgement

The authors would like to appreciate to the Science and Research Foundation of the Japanese Ministry of Education for financial supporting of this research (C 12650125).

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