I. Introduction

A dual diffraction grating grazing incidence interferometer for the measurement of artifacts with cylindrical geometry has been developed. [1-4] Figure 1 shows a schematic of this type of interferometer. A cylindrical test artifact produces annular fringes that, to first order utilizing geometrical raytracing, result from deviations of the sample from a true cylinder. This interferometric configuration has previously measured axial sinusoidal artifacts for determination of instrument spatial frequency response.[5] Since a periodic structure that produces repetitive changes in phase, amplitude, or both of an impinging wavefront can be considered as a diffraction grating [6] the effect of diffraction on the resultant fringe pattern was examined. Because any surface profile can be deconvolved into its Fourier components, a surface can be modeled as a sum of individual sinusoidal profiles with each spatial frequency introducing some degree of diffraction upon an incident light beam. An investigation was undertaken to determine how a second order approximation of diffraction from an axially sinusoidal artifact influence profile reconstruction.

II. Theoretical Analysis

Assume a single frequency, axially sinusoidal artifact of amplitude m/2 and wavelength d, where d is smaller than the length of the part, z₀, is tested by a diffractive grazing incidence interferometer (the derivation is the same for both planar and rotational symmetry parts). An example of this is shown in Figure 2.
Figure 2: Diagram demonstrating diffraction from a cylindrical test artifact for a dual grating grazing incidence interferometer. The measurement artifact acts as a reflective phase grating on the impinging wavefront diffracting the reflected light. Employing plane wave diffraction theory [7], the field $U$ as a function of position $x$ on the second grating for a single frequency phase grating, keeping only the zeroth and first diffracted orders from the part and neglecting aperture effects, is

$$U(x) = J_0 \left( \frac{m}{2} \right) + J_1 \left( \frac{m}{2} \right) \left( e^{ikx \sin \theta_{-1}} - e^{ikx \sin \theta_{+1}} \right)$$

where $J_0$ and $J_1$ are the zeroth and first order Bessel functions [8] respectively, $k$ is the wave number, and $\theta_{-1}$ and $\theta_{+1}$ are the $-1$ and $+1$ diffracted angles from the artifact with respect to the artifact surface normal respectively. Inserting the transformation $\tan \theta_0 = x / z$ and a first order Taylor series expansion of $\sin \theta_{-1}$ and $\sin \theta_{+1}$ into equation 1, the field as a function of the $z$ axis along the length of the measurement artifact becomes

$$U(z) = J_0 \left( \frac{m}{2} \right) + 2 J_1 \left( \frac{m}{2} \right) i \sin(k_d z)$$

with $k_d$ the profile spatial frequency and $z$ the axial position on the part. This form of $U(z)$ reproduces the single frequency axial sinusoid on the measurement artifact and is the expected first order approximation of measuring the artifact by the grazing incidence interferometer. Inserting a second order Taylor series expansion of $\sin \theta_{-1}$ and $\sin \theta_{+1}$ into equation 1 and again transforming to $z$ coordinates, the field now becomes
where \( \alpha \) is the factor \( g^2/d\lambda \) with \( g \) being the pitch of grating 1 and 2 (assuming symmetric grating pitch). To a second order approximation of \( \sin \theta_{1,1} \), there is an enveloping function distorting the artifact profile reconstruction. For a part with a total length of 30 mm and an axial wavelength \( d \) of 3 mm with \( g \) being a nominal 8 microns and a \( \lambda \) of 0.6328 microns, \( \alpha k_d \) is approximately 0.02\( k_d \). The enveloping function has a wavelength about 50 times longer than the part profile. Depending on the spatial frequency of the axial profile and the position of the sinusoid along the envelope (at a peak, valley, or midpoint), the amplitude of the axial profile is a dependent on the position of the enveloping function. Generally, if we include \( n \) diffractive orders from the part, the field of the reconstructed profile, transformed to part coordinates can be written as

\[
U(z) = J_0 \left( \frac{m}{2} \right) + 2 \sum_{n=1}^{\infty} J_n \left( \frac{m}{2} \right) \left[ \cos \left( \frac{n \alpha}{2} \right) \right] \left[ \sin (n k_d z) \right]
\]

The enveloping wavelength is a function of \( n^2 \) but also a function of \( J_n \) which approaches zero very fast for small values of \( m/2 \). Values of \( m/2 \) are less than 0.01 for form measurements of many parts tests by this interferometer. For a surface profile that can be decomposed into \( p \) sinusoids along the direction of the optical axis, the field of the reconstructed profile normalized to the zeroth order, can be shown to be

\[
U(z) = 1 + \sum_{p=1}^{p} \left[ J_1 \left( \frac{m}{2} \right) \right] \left[ e^{i k_p (\frac{\alpha p}{2}) z} \right] \left[ 2 i \sin k_p z \right]
\]

neglecting intermodulation and higher order terms that are much smaller than those listed in the equation above. Each of the previous calculations assumed the focus position of the imaging system is the position of the second grating. Transforming the focus position to a position \( z' \) between grating 1 and grating 2, the field along the axis of the part can be written as

\[
U(z) = \left[ e^{i k z'} \right] \left[ J_0 \left( \frac{m}{2} \right) + 2 J_1 \left( \frac{m}{2} \right) \left[ e^{i k_d \left( \frac{\alpha}{2} \right) (z-z' \left( 1-\frac{\alpha^2}{4} \right))} \right] \left[ \sin \left( k_d \left( z-z' \frac{\alpha^2}{2} \right) \right) \right]
\]

The focus shift has the effect of changing the position of the enveloping function by an amount \( z' \alpha/2 \) (to first order) and to a lesser extent changing the position of the axial sinusoid by a factor \( z' \alpha^2/2 \) (this effect is second order). The change in field for multiple frequency parts is analogous to the change for single frequency components. Since the position of the enveloping function can be changed, multiple measurements at different focal positions allow the calculation of the
enveloping function wavelength for each spatial frequency. This would enable correction for second order diffractive distortions of an artifact measurement.

III. Conclusions

Measurement of axially sinusoidal artifacts by a dual diffractive grating grazing incidence interferometer was theoretically described to a second order approximation. An enveloping function of the original profile was predicted. The phase of the focal plane was shown to change as a function of the imaged focal plane. A compensation method to correct for the second order distortion was presented. Future work involves verifying the existence of the second order enveloping function and the ability to change its position by varying the imaged focal plane.

IV. References

2. ATP Project Brief 95-01-0022, “Non-contact optical metrology of complex surface forms for precision industrial manufacturing,” General Competition (July 1995).