A novel practical approach to cylindricity evaluation: An experimental report *

G. Singh, † O. Devillers, ‡ and F. P. Preparata §

Evaluating cylindricity is a very important application in metrology, since cylindrical surfaces are ubiquitous in industrial machining. Cylindricity is normally evaluated by measuring radial form (in sections normal to the nominal axis), or axial form (along nominal generatrices), or a combination of the two. In this note, we focus on radial form measurements. Reliable measures of the quality of a cylindrical surface therefore is of paramount importance, essentially to avoid overly conservative criteria that may lead to costly unnecessary rejection of surfaces perfectly complying with specifications. Cylindricity, a single parameter with the physical dimension of a length, is this measure of quality.

The definition of cylindricity that best conforms with the standards ISO 1101 and DIN 7184 derives from the notion of minimum zone cylinder, i.e., the region of space contained between two co-axial cylinders with minimum radial separation containing all the data points. Cylindricity is the radial separation of a minimum zone cylinder. Unfortunately, the construction of the zone cylinder is a much more difficult geometric problem than the better understood construction of the zone circle in two dimensions. There are several reasons for the mentioned difficulty. The minimum zone cylinder is, in general, defined by six points, but there may be several 6-point subsets of the point set yielding minimum zone cylinders; in addition, given six points, it can be shown that constructing the zone cylinder through these points involves the solution of a system of six degree-4 equations, a task requiring

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† Mahr Federal, Providence, RI
‡ INRIA, BP93, 06902 Sophia-Antipolis, France.
§ Computer Science Department, Brown University, Providence, RI 02912, USA
sophisticated algebraic geometry tools. Due to this substantial geometric and algorithmic difficulty, it is obviously a frequent metrological practice to resort to simpler, but less effective, surrogates, such as reduction of cylindricity to circularity, or optimizations based on least square fits. Reductions to circularity involve an estimate of the axis, projection of all data points on a plane normal to this axis, and the computation of the minimum zone-circle in this plane. Alternatively, the axis is estimated as the least-square-fit of the centers of a set of normal sections, and the optimization aims at obtaining a "small" perturbation of this axis which minimizes radial separation [Ch85]. Noteworthy is an iterative approximation method [CF95], which shares with our approach the use of linear programming but is based on basically different criteria. The same paper contains a good survey of the literature on the subject.

Recently a new method has been proposed [DP00] that, as a number of previous optimization approaches, avoids the direct construction of the zone cylinder of a point set, and approximates it with controlled accuracy through a computationally very efficient process. The approach is based on the following idea. A cylinder of near-vertical axis is replaced (approximated) by a one-sheet hyperboloid with the same axis and such that its sections in planes normal to the z-axis are circles. Correspondingly, a zone cylinder is replaced by a zone hyperboloid, analogously defined. Clearly, the cylinder and its associated hyperboloid are two distinct geometric objects, but if their common axis is brought to coincide with the z-axis by a rigid motion, the hyperboloid tends to a cylinder and in fact cylinder and associated hyperboloid become provably the same object. This property is the key to the approach, since, rather than the set of zone cylinders, we perform the much easier search of the set of zone hyperboloids. The latter task is a computationally much simpler, since it is implemented as a linear programming in six dimensions and runs in time nearly proportional to the size of the point set. Each measured point gives rise to two linear constraints, corresponding to lying on the correct sides of the inscribed and the circumscribed hyperboloids, and the objective function is the radial separation. Since the LP-optimization also returns the axis of the zone hyperboloid (as an offset with respect to the origin and a direction), the values of the axis parameters are an excellent measure of the quality of the approximation.

As it normally happens, the axis of the minimum zone cylinder is not vertical, because both of the imperfections of the artifact being evaluated and of the misalignment of the measuring apparatus. Our objective is the
determination of this axis. Therefore, we take the axis of the computed zone hyperboloid as a first approximation to it, and subject the point set to a rigid motion bringing the $z$-axis to coincide with the axis of the hyperboloid. By this coordinate transformation the (unknown) axis of the minimum zone cylinder is brought closer to the $z$-axis than it was initially. Iterating this process, we can approach the cylinder axis with excellent precision. Typically, 2-3 iterations obtain a satisfactory solution of the original problem. The theory of the method was presented in detail in [DP00]; the purpose of this paper is to corroborate the theory with actual metrological experiments on physical objects.

We shall refer to the common metrological set-up where the test object is positioned with nominally vertical axis on a rotating platform and a probe takes samples in chosen horizontal planes. For each specimen, the measurement data consist of 256 angularly evenly spaced points on three evenly spaced horizontal slices. Each data point is measured using an off-the-shelf commercial instrument, which yields a triplet $(R, \theta, z)$, of radius, angle, and elevation. This data, converted into cartesian form constitutes the input to the LP-optimization. The axis information returned by the LP-algorithm is used to change the frame of reference (by identifying the obtained axis with the $z$-axis), and the procedure is iterated on the modified data set.

We have performed experiments on an large set of test parts, of which we shall report below results on the following typical examples:

1. part 1: a master disc of radius 0.625in, a high precision object.

2. part 2: an aluminum bar stock, a very coarse object.

3. part 3: a brass bushing, a medium quality object damaged by two grooves on opposite sides.

4. part 4: a cylindrical spacer, slightly tapered in the middle section.

For each of these four objects we report below the following data: the nominal radius (in inches), the cylindricity as the deviation from the least-square cylinder (LS-method), the cylindricity as the minimum-zone optimization starting from a least-square axis (LS-MZC), the cylindricity as obtained by the hyperboloid method (Hyperb.), the number of performed iterations, and a measure of quality expressed the the "relative offset", which is defined as the sum of the offsets of the centers at the three section normalized by the
radius. Cylindricities are expressed in μinches.

<table>
<thead>
<tr>
<th>object</th>
<th>radius</th>
<th>LS - method</th>
<th>LS - MZC</th>
<th>Hyperb.</th>
<th>iterat.</th>
<th>offset</th>
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<td>52.22</td>
<td>2</td>
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References

