The Control of Six Degree-of-Freedom Magnetically Suspended Stage

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Abstract: This paper describes the controller design for a magnetically suspended stage with six degrees-of-freedom. The stage employs a single platen as the moving part with a large travel volume of 25mm × 25mm × 0.1mm. We will present the modeling of the platen taking into account motion couplings between the different movements, and a controller design that decouples the six-input six-output control into six separate single-input single-output controls. Results showing 0.3nm stepping performance are given.

1. Introduction

Fine motion stages are widely used in scanning probe microscopes. Magnetically suspended stages used for this purpose are a development of recent years[1-4]. They can possess the advantage of both long travel and high resolution. M. Holmes has developed a prototype of such a stage [5,6]. This paper addresses control related problems on the stage. It begins with an introduction of the stage, then the modeling and controller design for the stage. Finally a 0.3 nm step is shown.

2. The magnetically suspended stage

Figure 1 shows the magnetically suspended stage with some parts moved aside and some eliminated to show the key inside structure. The stage is composed of four linear motors, a floating platen and a metrology target and sensors etc. Dow Corning 200 fluid with viscosity of 10,000 cs is filled between the platen and the frame. The platen is designed to neutrally float in the fluid. Four linear motors provide the necessary force to suspend and servo the platen, which has a travel volume of 25mm×25mm×0.1mm. The metrology target is attached on the top of the platen and extends out of the fluid. Its positions are monitored by the three interferometers and three capacitive gauges. The stage is mounted on an air isolated optical table. The whole system is placed in a temperature controlled metrology lab.

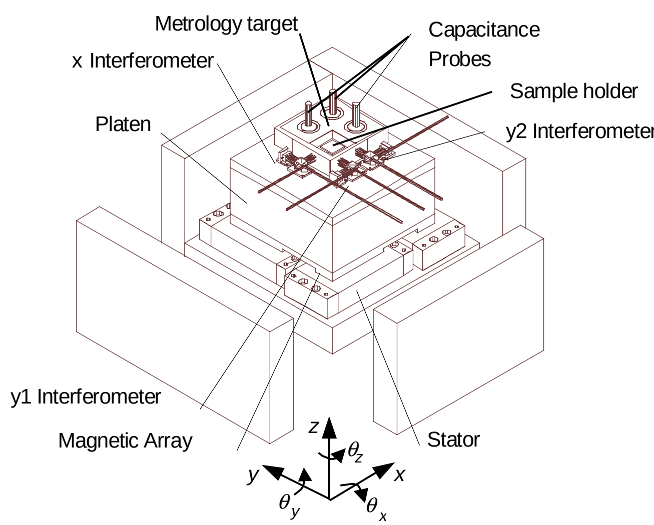

Figure 1 Magnetically suspended stage with the front frames moved aside, sensor mounts eliminated

3. Modeling

Fine motion control depends on accurate modeling. The stage has only one moving part – the platen as shown in figure 1. All movements in x, y, z, θ_x, θ_y and θ_z are realized by servoing the platen. The platen can be modeled as

$$\begin{bmatrix} x(s) \\ y(s) \\ z(s) \\ \theta_x(s) \\ \theta_y(s) \\ \theta_z(s) \end{bmatrix} = \begin{bmatrix} H_x(s) & 0 & 0 & 0 & H_{\theta_y x} & H_{\theta_z x} \\ 0 & H_y(s) & 0 & H_{\theta_x y} & 0 & H_{\theta_z y} \\ 0 & 0 & H_z(s) & H_{\theta_x z} & H_{\theta_y z} & 0 \\ 0 & 0 & 0 & H_{\theta_x}(s) & 0 & 0 \\ 0 & 0 & 0 & 0 & H_{\theta_y}(s) & 0 \\ 0 & 0 & 0 & 0 & 0 & H_{\theta_z}(s) \end{bmatrix} \begin{bmatrix} F_x(s) \\ F_y(s) \\ F_z(s) \\ T_{\theta_x}(s) \\ T_{\theta_y}(s) \\ T_{\theta_z}(s) \end{bmatrix} \tag{1}$$

where s is Laplace transformation factor, $x(s)$, $y(s)$, $z(s)$, $\theta_x(s)$, $\theta_y(s)$ and $\theta_z(s)$ are the positions of the platen in x, y, z, θ_x, θ_y and θ_z axes respectively, $F_x(s)$, $F_y(s)$, $F_z(s)$, $T_{\theta x}(s)$, $T_{\theta y}(s)$ and $T_{\theta z}(s)$ are the forces or torques applied to the platen in x, y, z, θ_x, θ_y and θ_z axes respectively, $H_i(s)$ ($i = x, y, z, \theta_x, \theta_y$ and θ_z) are the transfer functions of the platen in each separate degree-of-freedom respectively, and $H_{ij}(s)$ ($i = \theta_x, \theta_y, \theta_z$ and $j = x, y, z$) are the coupling transfer functions from the torques that are applied for angular movements to linear motions. Motion couplings occur when the rotary axes do not pass the point where the test sample is located. In this stage, the linear motors are placed at the bottom of the platen while the test sample is located at one corner of the metrology target (see figure 1). None of the rotary axes passes the sample. Whenever there is an torque applied to the platen, there will be an accompanied motion in *x, y* or *z,* as well as the desired angular motion.

4. Digital controller design

The stage is a typical multi-input multi-output system. However, to avoid making the controller very complex, it is preferable to decouple the six-input six-output stage to six single-input single-output controls. Please note that the coupling motions occur only from angular to linear ones, but not visa versa. Therefore angular controllers can be designed independently. Once they are determined their coupling motions to linear ones are also definite or predictable. Then the linear motion controller can be designed to take into account the couplings from the angular controls. Digital controllers employing state design are used here[7]. For the convenience of mathematic calculation, it is assumed that angular controllers use angular position and velocity as their describing states while linear controllers use linear position and velocity as their states.

(1) θ_z controller

The model of the platen in θ_z can be described in state space as

$$\boldsymbol{\theta}_z(k+1) = \boldsymbol{\Phi}_{\theta_z}\boldsymbol{\theta}_z(k) + \boldsymbol{\Gamma}_{\theta z}T_{\theta z}(k) \tag{2}$$

$$\theta_z(k) = \mathbf{H}\boldsymbol{\theta}_z(k) \tag{3}$$

where $\boldsymbol{\theta}_z = \begin{bmatrix}\theta_z & \dot{\theta}_z\end{bmatrix}^T$, $\mathbf{H} = \begin{bmatrix}1 & 0\end{bmatrix}$, $\boldsymbol{\Phi}_{\theta_z}$ and $\boldsymbol{\Gamma}_{\theta z}$ are the matrices representing the platen transfer function in the discrete domain, and they are equivalent to the transfer function $H_{\theta_z}(s)$, k is the sequence number of control cycles, $T_{\theta z}$ is the servoing torque applied for θ_z motion. The control diagram for θ_z is shown in figure 2. To eliminate the residual error between the input and output, an integrator $\theta_{zI}(k)$ is necessary, which is

$$\theta_{zI}(k+1) = \theta_{zI}(k) + \theta_z(k) - \theta_{zd}(k) = \theta_{zI}(k) + \mathbf{H}\boldsymbol{\theta}_z(k) - \theta_{zd}(k) \tag{4}$$

where θ_{zd} is the desired position and θ_z is the actual position. Augmenting the integrator, model (2) becomes

$$\begin{bmatrix}\theta_{zI}(k+1) \\ \boldsymbol{\theta}_z(k+1)\end{bmatrix} = \begin{bmatrix}1 & \mathbf{H} \\ 0 & \boldsymbol{\Phi}_{\theta_z}\end{bmatrix}\begin{bmatrix}\theta_{zI}(k) \\ \boldsymbol{\theta}_z(k)\end{bmatrix} + \begin{bmatrix}0 \\ \boldsymbol{\Gamma}_{\theta_z}\end{bmatrix}T_{\theta_z}(k) - \begin{bmatrix}1 \\ \mathbf{0}\end{bmatrix}\theta_{zd}(k) \tag{5}$$

The control law, following equation (5), is

$$T_{\theta_z}(k) = -\begin{bmatrix}K_{\theta_z I} & \mathbf{K}_{\theta_z}\end{bmatrix}\begin{bmatrix}\theta_{zI}(k) \\ \boldsymbol{\theta}_z(k)\end{bmatrix} + \mathbf{K}_{\theta_z}\mathbf{N}\theta_{zd}(k) \tag{6}$$

where $[K_{\theta_z I} \quad \mathbf{K}_{\theta_z}]$ is the gain matrix, which can be determined using the *Matlab* function *acker* based on the coefficient matrix in equation (5), **N** is a matrix to convert the input θ_{zd} to a reference state, $\mathbf{N}=[1,0]^T$. The estimator is to give angular velocity information that is not measured directly by the interferometers and capacitive gauges.

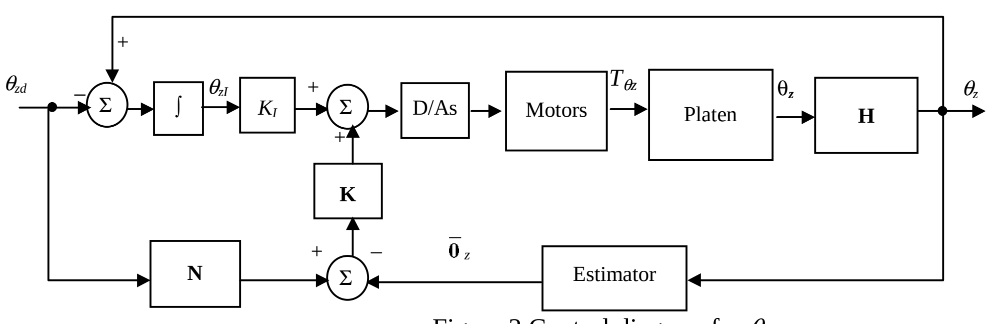

Figure 2 Control diagram for θ_z

(2) θ_y controller

Similar to that of θ_z, the model of θ_y can be described as

$$\boldsymbol{\theta}_y(k+1) = \boldsymbol{\Phi}_{\theta_y}\boldsymbol{\theta}_y(k) + \boldsymbol{\Gamma}_{\theta y}T_{\theta y}(k) \tag{7}$$

$$\theta_y(k) = \mathbf{H}\boldsymbol{\theta}_y(k) \tag{8}$$

where $\mathbf{\Phi}_{\theta_y}$ and $\mathbf{\Gamma}_{\theta y}$ are the matrix coefficients equavalent to the transfer function $H_{\theta_y}(s)$, $T_{\theta y}$ is the torque applied for θ_y motion. Augmenting a similar integrator θ_{yI} to model (7) results

$$\begin{bmatrix}\theta_{yI}(k+1)\\ \mathbf{\theta}_y(k+1)\end{bmatrix}=\begin{bmatrix}1 & \mathbf{H}\\ 0 & \mathbf{\Phi}_{\theta_y}\end{bmatrix}\begin{bmatrix}\theta_{yI}(k)\\ \mathbf{\theta}_y(k)\end{bmatrix}+\begin{bmatrix}0\\ \mathbf{\Gamma}_{\theta_y}\end{bmatrix}T_{\theta_y}(k)-\begin{bmatrix}1\\ \mathbf{0}\end{bmatrix}\theta_{yd}(k) \tag{9}$$

where θ_{yd} is the desired angular position. The control law, following equation (9), is

$$T_{\theta_y}(k)=-\begin{bmatrix}K_{\theta_{yI}} & \mathbf{K}_{\theta_y}\end{bmatrix}\begin{bmatrix}\theta_{yI}(k)\\ \mathbf{\theta}_y(k)\end{bmatrix}+\mathbf{K}_{\theta_y}\mathbf{N}\theta_{yd}(k) \tag{10}$$

where $[K_{\theta_{yI}} \quad \mathbf{K}_{\theta_y}]$ is the gain matrix, which can also be determined using the *Matlab* function *acker* based on the coefficient matrix in equation (9), **N** is the matrix to convert input θ_{yd} to a reference state, $\mathbf{N}=[1,0]^T$.

(3) *x* controller

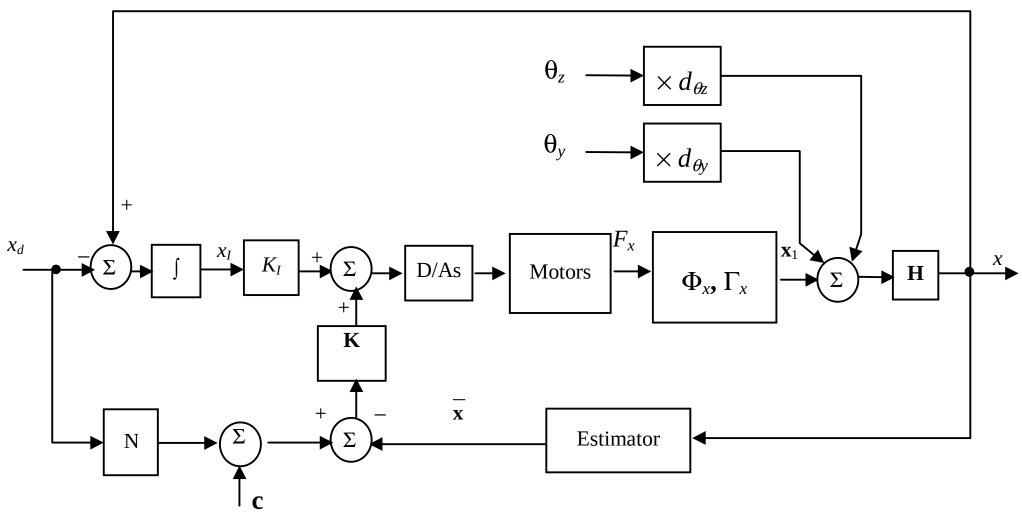

Figure 3 the *x* control

As far as the pure linear motion in *x* is considered, it has a similar model to that of angular motion, as

$$\mathbf{x}_1(k+1)=\mathbf{\Phi}_x\mathbf{x}_1(k)+\mathbf{\Gamma}_xF_x(k) \tag{11}$$

where Φ_x and Γ_x are the matrix representing transfer function $H_x(s)$ in state space. However the motion coupling from θ_y and θ_z to x must be considered for fine control. In this case the x controller can be modified as shown in figure 3. The new x motion model becomes

$$\mathbf{x}(k+1)=\mathbf{x}_1(k+1)+\mathbf{\theta}_y(k+1)d_{\theta_y}+\mathbf{\theta}_z(k+1)d_{\theta_z} \tag{12}$$

$$x(k)=\mathbf{Hx}(k) \tag{13}$$

where d_{θ_y} and d_{θ_z} are respective distances that couple angular motions in θ_y and θ_z into linear motion in x.

Combining equations (2), (7), (11) and (12) results in

$$\mathbf{x}(k+1)=\mathbf{\Phi}_x\mathbf{x}_1(k)+\mathbf{\Gamma}_xF_x(k)+\mathbf{\Phi}_{\theta_y}\mathbf{\theta}_y(k)d_{\theta_y}+\mathbf{\Gamma}_{\theta_y}T_{\theta_y}(k)d_{\theta_y}+\mathbf{\Phi}_{\theta_z}\mathbf{\theta}_z(k)d_{\theta_y}+\mathbf{\Gamma}_{\theta_z}T_{\theta_z}(k)d_{\theta_z} \tag{14}$$

Since $\mathbf{x}_1(k)=\mathbf{x}(k)-\mathbf{\theta}_y(k)d_{\theta_y}-\mathbf{\theta}_z(k)d_{\theta_z}$, equation (14) can be rewriten as

$$\mathbf{x}(k+1)=\mathbf{\Phi}_x\mathbf{x}(k)+\mathbf{\Gamma}_xF_x(k)+\mathbf{c}_x(k) \tag{15}$$

where $\mathbf{c}_x(k)=(\mathbf{\Phi}_{\theta_y}-\mathbf{\Phi}_x)\mathbf{\theta}_y(k)d_{\theta_y}+\mathbf{\Gamma}_{\theta_y}T_{\theta_y}(k)d_{\theta_y}+(\mathbf{\Phi}_{\theta_z}-\mathbf{\Phi}_x)\mathbf{\theta}_z(k)d_{\theta_z}+\mathbf{\Gamma}_{\theta_z}T_{\theta_z}(k)d_{\theta_z}$. $\mathbf{c}(k)$ is the motion coupling from $\theta_y(k)$ and $\theta_z(k)$ to $\mathbf{x}(k+1)$. To eliminate the residual error between the actual position x and desired position x_d, an integrator is also included

$$x_I(k+1)=x_I(k)+x(k)-x_d(k)=x_I(k)+\mathbf{Hx}(k)-x_d(k) \tag{16}$$

and arriving at the augmented model

$$\begin{bmatrix} x_I(k+1) \\ \mathbf{x}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{H} \\ 0 & \mathbf{\Phi}_x \end{bmatrix} \begin{bmatrix} x_I(k) \\ \mathbf{x}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{\Gamma}_x \end{bmatrix} F_x(k) + \begin{bmatrix} -x_d(k) \\ \mathbf{c} \end{bmatrix} \quad (17)$$

from which the corresponding control law is obtained

$$F_x(k) = K_I x_I - \mathbf{Kx}(k) + \mathbf{K}(\mathbf{N}x_d - \mathbf{c}) \quad (18)$$

(4) θ_x, y and z control

θ_x controller has a similar mathematical form as the θ_y and θ_z controllers except some variables are replaced with their θ_x counterparts. The y and z controllers have similar forms as x except the corresponding variables are replaced with their counterparts.

5. Step response

With the new model and controller, stable motions and positioning are observed. Figure 4 shows the 0.3nm step response of the stage.

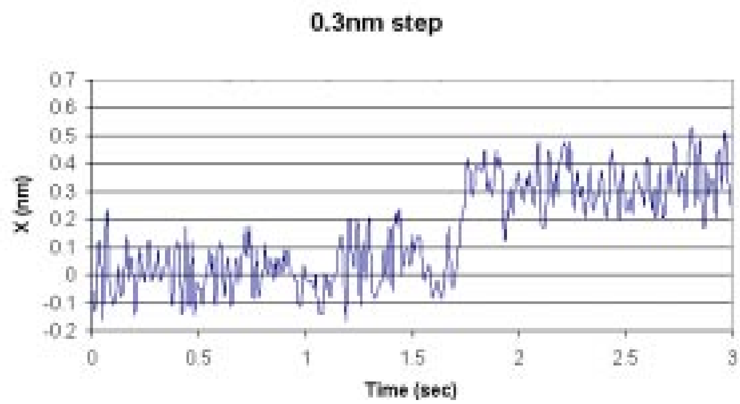

Figure 4 0.3 nm step

6. Conclusion

The modeling and control of the platen is investigated with consideration of the motion couplings. The couplings occur from angular motions to linear motions, but not in reverse direction. This phenomenon forms the basis for decoupling the six-input six-output platen control system into six separate single-input single-output controls. The angular controllers are designed independently, and the linear controllers are designed taking into account the couplings from angular motions.

Acknowledgments

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