

Hybrid Simulation and Experimental Method of Uncertainty Analysis

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1. Introduction

According to the scope of the *Guide to the Expression of Uncertainty in Measurement* (GUM) [1], the general rules for evaluating and expressing uncertainty that are established in the GUM are for various fields and levels of accuracy. The GUM specifically states that its principles are designed to cover a broad range of measurements including shop floor and production measurements, laboratory calibrations, and applied research and development measurements. The measurement uncertainty examples found in the GUM and in the published literature, however, are generally applicable only to calibrations and advanced measurement techniques. Little practical guidance is available for the various common industrial dimensional measurements that are found in places such as on production shop floors and in quality assurance laboratories.

In this paper, a general methodology for uncertainty analysis is presented that combines simulation and experimental techniques and is in accordance to the principles of the GUM. The hybrid technique brings together the concepts of measurement uncertainty simulation [2,3,4] with industrial developed practices in experimentally testing the repeatability of production measurement processes [5]. The developed approach is particularly applicable to situations where individual gages or measuring instruments are used for a variety of measurement tasks, such as is common in production measurements. The hybrid technique is applicable from simple measurement tasks, such as measuring diameter with a micrometer, to complex tasks, such as measurements made with coordinate measuring machines (CMMs).

2. General Uncertainty Model for Independent Contributors

Following the philosophy of the GUM, a measurement is an attempt to estimate the value of a specific measurand. The lack of knowledge in the value of the measurand from a particular measurement process is expressed by an estimation of the uncertainty of the measurement. In accordance to the GUM, the uncertainty is represented by the combined standard uncertainty, $u_c(y)$, for a measurement result y . If all contributors to the uncertainty can be treated as independent, then the combined standard uncertainty can be determined by the positive square root of

$$u_c^2(y) = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \quad (1)$$

where $u(x_i)$ represents the standard uncertainty of any one of the N independent quantities that contribute to the uncertainty of the measurement, and partial derivatives $\partial f / \partial x_i$ are the sensitivity coefficients that relate the change of the output estimate, the measurement result y , to the input estimates x_i . The governing equation for the measurement process is defined by

$$y = f(x_i) \quad \text{for } i = 1 \text{ to } N \quad (2)$$

To estimate the combined standard uncertainty, it is necessary to best determine the quantities influencing the measurement result, estimate the standard uncertainties associated with the contributors, and calculate the appropriate sensitivity coefficients. In estimating the standard uncertainties, the GUM recognizes two

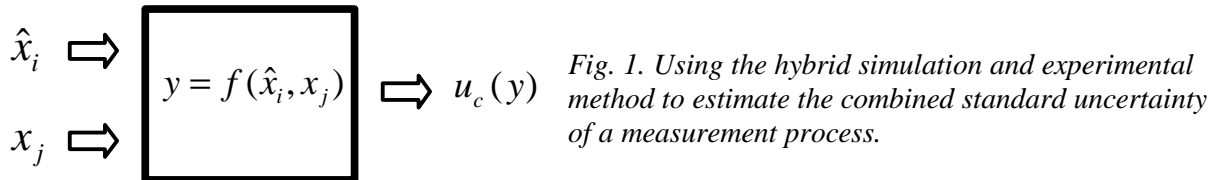
separate types, those evaluated by statistical methods (Type A), and those evaluated by other means, such as the application of sound scientific or engineering judgment (Type B). In practice, many uncertainty analyses use a combination of Type A and B techniques [1,6]. In the hybrid approach introduced in this paper, Type A and B estimates are not formally used, but rather the combined standard uncertainty of a measurement is determined directly through a combination of simulation and experimental techniques.

3. Description of the Hybrid Approach

In the hybrid approach, a Type A statistical experiment is specially developed such that particular input estimates, x_j , are allowed to influence the measurement result while the other independent input estimates have negligible effect. These other input estimates are then estimated using a Monte Carlo simulation technique and are labeled, \hat{x}_i , to distinguish them from x_j . The equation of the measurement process can therefore be written in the form

$$y = f(\hat{x}_i, x_j) \quad \text{for } i, j = 1 \text{ to } N \text{ and } i \neq j \quad (3)$$

where y is again the output estimate and \hat{x}_i and x_j represent estimates of all the influence quantities determined to be significant to the measurement process. Using the governing equation for the measurement process in the form of Eq. (3), the simulated input estimates \hat{x}_i are then numerically inserted into the independent statistical experiment that is dependent only on x_j . In this manner the uncertainty due to the \hat{x}_i are propagated into the Type A experiment that was originally only sensitive to the x_j . A statistical analysis of this hybrid experiment is then used to directly estimate the combined standard uncertainty of the measurement result y . This process is shown graphically in Fig. 1. The hybrid approach is motivated by situations where it is more economical and practical to develop Type A experiments that are sensitive to specific independent contributors instead of using Type B knowledge or where it is difficult to directly determine the appropriate sensitivity coefficients of Type B estimates of standard uncertainties.



4. Hybrid Uncertainty Method Applied to a CMM

To demonstrate the usefulness of the hybrid method for estimating the uncertainty of complex measuring systems, the method is applied to a CMM process. Though there is no well-accepted method for estimating the uncertainty of CMM results, there is literature available that discusses the errors in measurements made with CMMs [7,8], and standards are available for testing their performance [9,10]. The difficulty lies in transposing the general knowledge of CMM capabilities into uncertainty estimates for particular measurement tasks. This problem is compounded by the fact that CMMs are used for a wide variety of measurements and that these measurements can be made in many different manners. While information regarding single point measuring uncertainty may be available, how that uncertainty propagates into the myriad of potential coordinate measurement processes is quite challenging.

The hybrid approach is implemented by developing a measurement experiment that is sensitive to particular x_j . Monte Carlo simulation techniques are then used to propagate additional input estimates,

\hat{x}_i , into the experiment assuming that the experiment was previously not sensitive to the \hat{x}_i . For CMMs, the hybrid approach is particularly useful if the simulation inputs can be generated within the CMM software and propagated into actual measurement data in-situ. This is the approach that was developed and is discussed here.

4.1 Generating and Propagating Simulation Values In-situ

The majority of commercially available CMM software is quite powerful and resides on advanced computing platforms. Using the features within the CMM software it is often possible to develop the necessary code to generate the Monte Carlo simulation values in real time during CMM operation. In addition, it is also necessary to be able to propagate the simulation values into the actual measurement results. The approach taken here is to simulate coordinate point measuring errors and then propagate them into the actual measured points. This is done on a point-by-point basis for each coordinate prior to the calculation and evaluation of the measured geometry. In this manner, the measurement uncertainty represented in the simulation process is propagated through the measurement algorithms of the CMM software.

The process of generating and propagating the simulation values in-situ is shown in Fig. 2. For each actual measured coordinate point, (X_i, Y_i, Z_i) , the associated errors, $(\mathbf{d}K_i, \mathbf{d}L_i, \mathbf{d}Z_i)$, are generated by sampling from the appropriate estimated distribution. This process creates a set of new measurement points, (X'_i, Y'_i, Z'_i) , which are then evaluated by the CMM software according to the feature being measured. Fig. 2 shows an example of a circle diameter being measured; D' is the final calculated diameter of the circle, and the function f represents the algorithm used to calculate the circle diameter from the set of n data points. This process would be repeated a number of times according to how the experiment was designed, and the statistics for D' would be used to estimate the combined standard uncertainty for the measurement of the diameter.

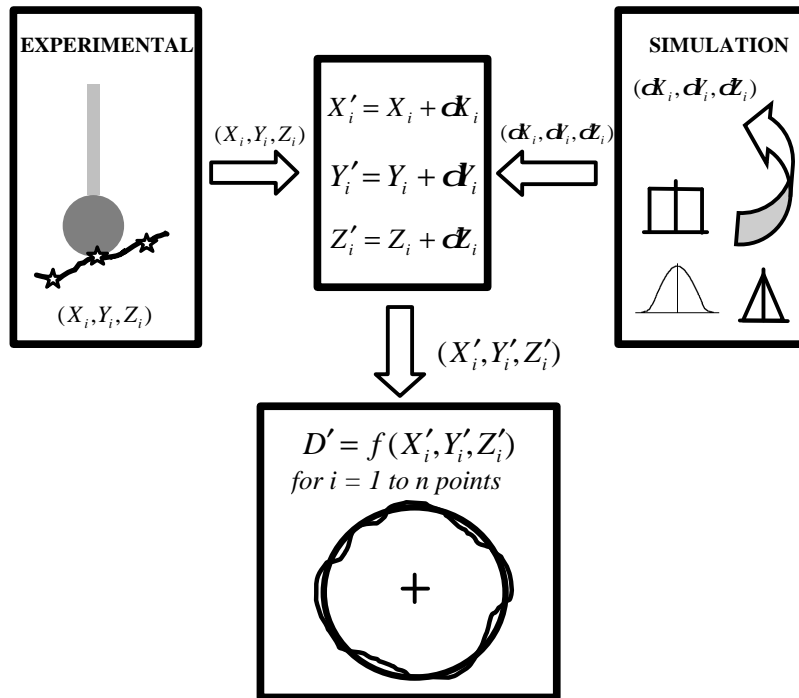


Fig. 2. In-situ generation and propagation of simulated values into the experimental results.

4.2 Hybrid Uncertainty Implementation on a CMM

The hybrid uncertainty technique is implemented on a CMM for the measurement of the diameter of a small bore. Based on performance test results, and for example purposes here, the standard uncertainty of the point measuring capability of the CMM is estimated to be 2 μm in each of the three axes directions. A Monte Carlo simulation is created within the measurement software so that each actual point is modified as discussed earlier and as shown in Fig. 2. For example purposes, a circle diameter is measured using three equally spaced points.

The bore on the actual workpiece is measured using three points and the least-squares circle fitting algorithm resident in the CMM software. A single part is measured 30 times. Between trials, the probe stylus is recalibrated using normal operating procedures. The program is also created to randomly change the exact locations of the measured points. The points are rotated around the bore and shifted slightly along the axis of the bore. The points are, however, always measured using an equally spaced sampling strategy. The standard deviation of the 30 measured diameters, which represents the combined standard uncertainty of the measurement process, is calculated.

Using the hybrid approach, the combined standard uncertainty is estimated to be 5.13 μm . For comparison, the results of the experiment are also processed without the use of the hybrid module enabled in the CMM software. These results, which are now simply a Type A study, are combined with the Type B estimate of the point measuring uncertainty and the sensitivity coefficient for circle measurements using three points [11]. Using this approach, the combined standard uncertainty is estimated at 4.87 μm . This value differs by about 5% from the result of the hybrid method; this difference is within expected statistical variation indicating the proper functionality of the hybrid approach. The hybrid approach has the advantage, however, of being able to be used for any type of CMM measurement.

5. Acknowledgments

This work was made possible by the research facilities available within the Center for Precision Metrology at the University of North Carolina at Charlotte. The author would like to thank Dr. Robert Hocken, the National Science Foundation, and all the industrial affiliate members who support the Center.

6. References

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