

# Real Time Estimation of Temperature Distribution in a Ball Screw System Using Observer and Adaptive Algorithm

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**Abstract :** So far empirical models have been mainly studied for real-time application of thermal error compensation scheme in machine tools. However, the empirical models are somewhat unreliable because they cannot estimate correctly for the untested conditions. This paper describes a new approach to estimate the unknown heat inputs and temperature distribution in real time based on an analytical model. A 1-dimensional heat transfer problem is modeled with state space equations. And then observer is designed to estimate the intensity of heat source and the whole temperature field in real time using frequency domain approach and low pass filter. Thermal process is time-variant because parameters in thermal process are the function of temperature. So, a recursive parameter identification of adaptive algorithm is used to enhance the estimation accuracy of temperature distribution as well as heat inputs. To verify the reliability of the proposed real-time estimator, heat inputs and temperatures are estimated by observer and with measurements from two points of a ball screw system in case of the presence of measurement noise.

**Keywords :** Adaptive Algorithm, Ball Screw System, Inverse Heat Conduction Problem(IHCP), Mode Reduction, Observer, Recursive Parameter Estimation, State Space Equation

## 1. Introduction

Thermal error is the most important factor that affects the accuracy of precision machine tools since thermal error comprises 40-70% of the workpiece error in precision machining[1]. Attention is paid to software error compensation with the increase of computer capability[2]. The accuracy is determined by the estimation accuracy of thermal error model. Some empirical models are used to relate temperature measurements to thermal deformation[3]. But it is a tedious task to obtain empirical error models and these models are not accurate enough, and are unreliable for untested inputs. As for numerical solutions, FEM and FDM methods are not appropriate to the real time compensation since they consume much computation time[4]. The previous research using IHCP is studied under assumption that confined to case that the reference model is known precisely[5].

In this paper, the temperature distribution of ball screw system is estimated with a few number of temperature measurements. First, thermal process is modeled as state space equations using modal analysis and mode reduction for a 1-dimensional heat transfer problem. For a real time estimation of heat inputs and temperatures, observer is designed using the concept of low pass filter. A recursive parameter identification is used to enhance the estimation accuracy for various thermal conditions. The application results to a ball screw show that the proposed method gives the solution to the real-time estimation of temperature distribution.

## 2. Modeling of heat transfer

### 2.1 Modal analysis

The governing equation and boundary conditions of 1-dimensional unsteady state heat conduction problem are given in general form by

$$L_{cond}[T(x,t)] + M \frac{\partial T(x,t)}{\partial t} = Q(x,t) \quad (1-a)$$

$$B_L[T(x,t)]_{x=0} = l(t), \quad B_R[T(x,t)]_{x=L} = r(t) \quad (1-b)$$

To change Eq. (1-b) into a homogeneous boundary condition, the solution of Eq. (1-a) is assumed as

$$T(x,t) = Q(x,t) + g(x)l(t) + h(x)r(t) \quad (2)$$

where,  $g(x)$  and  $h(x)$  are polynomials which make homogeneous boundary condition about  $Q(x,t)$ .

From Eq. (1) and (2), a nonhomogeneous differential equation is given by Eq. (3). If the solution of Eq. (3) is assumed as  $\Theta(x,t) = \mathbf{q}(x)\mathbf{h}(t)$ , an eigenvalue problem is acquired as Eq. (4)

$$L_{cond}[\mathbf{Q}(x,t)] + M \frac{\partial \mathbf{Q}(x,t)}{\partial t} = Q(x,t) - \{l(t)L[g(x)] + \dot{l}(t)Mg(x)\} - \{r(t)L[h(x)] + \dot{r}(t)Mh(x)\} \quad (3)$$

$$L_{cond}[\mathbf{q}(x)] - LM\mathbf{q}(x) = 0 \quad (4-a)$$

$$B_L[\mathbf{q}(x)]_{x=0} = 0, \quad B_R[\mathbf{q}(x)]_{x=L} = 0 \quad (4-b)$$

The solution of Eq. (4) yields an infinite set of eigenvalues  $\mathbf{I}_n$  and corresponding eigenfunctions  $\mathbf{q}_n(x)$ . When it is assumed that the solution of Eq. (3) is given by Eq. (5) based on expansion theorem, uncoupled ordinary differential equation set is acquired as Eq. (6).

$$\mathbf{Q}(x, t) = \sum_{n=1}^{\infty} \mathbf{q}_n(x) \mathbf{h}_n(t) \quad (5)$$

$$\dot{\mathbf{h}}_n(t) + \mathbf{L}_n \mathbf{h}_n(t) = N_n(t), \quad n=1, 2, \dots \quad (6)$$

where,  $N_n(t)$  is generalized heat input.

For the heat convection problems, the same procedure is applied to eigenvalues, eigenfunctions and uncoupled ordinary differential equation set.

## 2.2 State space equation by mode reduction

In Eq. (6), the mode reduction is required because it is inefficient and impossible to formulate the state space equation with infinite modes. A criterion is established to aid in the elimination of modes to be used in the solution. Firstly,  $s_N$  can be defined in the following way

$$s_N^2 = \sum_{i=1}^N \left( \frac{\mathbf{q}_i(a_H)}{\mathbf{L}_i} \right)^2 \quad (7)$$

where  $N$  is the number of modes used in the approximation.  $N$  increases until Eq. (8) becomes violated.

$$\frac{s_N}{s_{N+1}} \geq \mathbf{b}, \quad 0 < \mathbf{b} < 1 \quad (8)$$

where,  $\mathbf{b}$  is the proportion of  $s_N$  and  $s_{N+1}$ , which define relative accuracy of results. After  $N$  is determined, the state space equation [Eq. (9)] is composed with a finite number of selected modes. If the sensor location is  $b_T$  ( $0 \leq b_T \leq L$ ), output  $y(t)$  is given by Eq. (10)

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) - \mathbf{D}\mathbf{z}(t) \quad (9)$$

$$y(t) = T(b_T, t) = \mathbf{C}\mathbf{x}(t) + \mathbf{E}\mathbf{z}(t) \quad (10)$$

where,

$$\mathbf{A} = \begin{bmatrix} \swarrow & & & \\ & -\mathbf{L} & & \\ & & \searrow & \\ & & & \ddots \end{bmatrix} = \begin{bmatrix} -\mathbf{L}_1 & & & \\ & -\mathbf{L}_2 & & \\ & & \ddots & \\ & & & -\mathbf{L}_N \end{bmatrix}$$

$$\mathbf{B} = [\mathbf{q}_1(a_H) \quad \mathbf{q}_2(a_H) \quad \dots \quad \mathbf{q}_N(a_H)]^T$$

$$\mathbf{C} = [\mathbf{q}_1(b_T) \quad \mathbf{q}_2(b_T) \quad \dots \quad \mathbf{q}_N(b_T)]$$

$$\mathbf{D} = \begin{bmatrix} \leftarrow \mathbf{D}_1 \rightarrow \\ \leftarrow \mathbf{D}_2 \rightarrow \\ \vdots \\ \leftarrow \mathbf{D}_N \rightarrow \end{bmatrix} = \begin{bmatrix} G_1^* & G_1 & H_1^* & H_1 \\ G_2^* & G_2 & H_2^* & H_2 \\ \vdots & \vdots & \vdots & \vdots \\ G_N^* & G_N & H_N^* & H_N \end{bmatrix}$$

$$\mathbf{E} = [g(b_T) \quad 0 \quad h(b_T) \quad 0]$$

$$\mathbf{x}(t) = \{\mathbf{h}_1(t) \quad \mathbf{h}_2(t) \quad \dots \quad \mathbf{h}_N(t)\}^T, \quad u(t) = q(t)/k$$

$$\mathbf{z}(t) = [l(t) \quad \dot{l}(t) \quad r(t) \quad \dot{r}(t)]^T$$

## 3. Observer design

In general, estimation of heat input is a focus in IHCP. The objective of this paper is to estimate the thermal deformation with the temperature field in real-time. The estimation of temperature distribution is important as well as the one of heat input.

The IHCP can be interpreted as a frequency-domain transfer system relating the estimated heat flux  $\hat{U}$  to the true heat flux  $U$  and the measurement noise  $N$ ,

$$\hat{U}(s) = G_U(s)U(s) + G_N(s)N(s) \quad (11)$$

To derive the relationship between  $G_U$  and  $G_N$ , consider the structure of the IHCP as depicted in the block diagram of Fig. 1. Measurement signal  $Y^*$  is given as

$$Y^* = Y + N = G_H Y + N \quad (12)$$

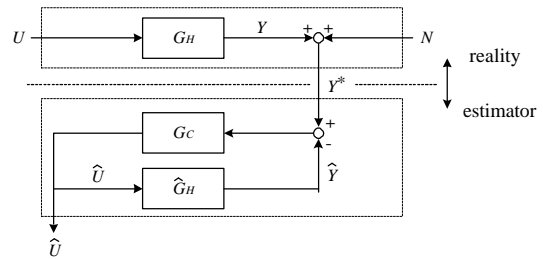


Fig. 1 Block diagram of IHCP solution

A reference model  $\hat{G}_H$  was assumed to be time-invariant and known precisely in the previous research[5]. However, Thermal process is time-variant in reality because parameters in thermal process ,e.g., heat convection coefficient, thermal diffusivity coefficient, are the function of temperature. So,  $\hat{G}_H$  is estimated recursively and  $G_C$  is updated by adaptive algorithm in this paper. The closed-loop transfer function of the feedback loop implemented in the estimator,

$$\hat{U} = \frac{G_C}{1 + G_C \hat{G}_H} Y^* \quad (13)$$

characterizes the behavior of the solution algorithm.  $G_U$  and  $G_N$  are found by combining Eqs. (14) and (15). Finally,  $\hat{U}$  is given as Eq. (16)

$$G_U = \frac{G_C \hat{G}_H}{1 + G_C \hat{G}_H} \quad G_N = \frac{G_C}{1 + G_C \hat{G}_H} \quad (14)$$

$$G_N = G_U \hat{G}_H^{-1} \quad (15)$$

$$\hat{U} = G_N(s) Y^*(s) \quad (16)$$

The amplification of measurement noise  $|G_N|$  for a given passband of the signal transfer function  $G_U$  can be minimized by maximizing the roll-off of  $|G_U|$  [5]. Therefore, we formulate a transfer function  $G_U$  satisfying the desired filtering properties by Butterworth analog filter design. The adaptive algorithm is applied to the observer to estimate the time-variant thermal parameter as shown in Fig. 2. It is based on the error reduction of the estimated temperature at another point.

#### 4. Application to ball screw

Using the proposed method, the temperature distribution of a ball screw system is estimated. The schematic diagram of an actual ball screw system is shown in Fig. 3. Table 1 shows the specification of the ball screw.

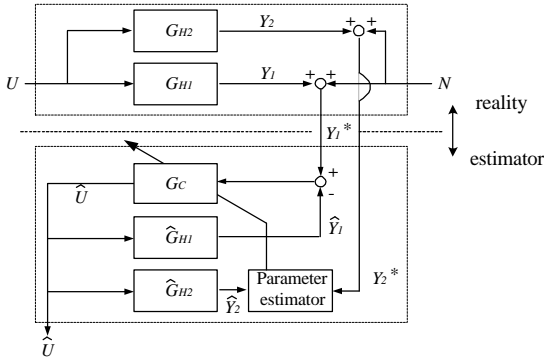


Fig. 2 Block diagram of proposed IHCP solution

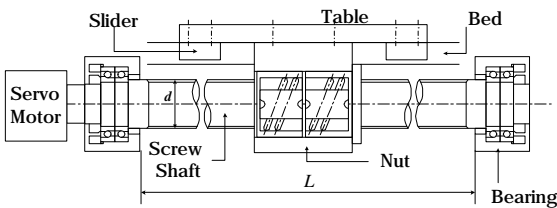


Fig. 3 Schematic diagram of a ball screw [6]

Table 1 Specification of ball screw [6]

Material	SCM415H
Diameter ( $d$ )	$40 \times 10^{-3}$ m
Screw length ( $L$ )	0.5 m
Heat capacity ( $C$ )	$5700 \text{ J/kg} \cdot \text{K}$
Thermal diffusivity ( $\alpha$ )	$1.67 \times 10^{-5} \text{ m}^2/\text{s}$
Heat conductivity ( $k$ )	$60.5 \text{ W/m} \cdot \text{K}$

The nut traverses a specific zone and the shaft rotates at a constant speed. In this case, boundary conditions and heat inputs can be assumed as Fig. 4. It is assumed that the boundary condition at motor side is an insulation condition, and that the boundary condition at the opposite side of motor is a convection condition. The heat input generated by the nut is assumed as a distributed heat input at the specific zone and the heat inputs at the bearings are assumed as concentrated heat inputs. When the nut traverses between  $a_1$  and  $a_2$ , the distributed heat input is given by Eq. (17) which is the modified equation of Otsuka's [6]. From the experiments by Kakino [7], it is shown that the heat inputs generated by friction at the bearings are proportional to the heat input intensity generated at the nut. Consequently, it can be assumed that the thermal behavior of the ball screw system is SISO system.

$$q_{nut}(t) = \frac{n \cdot P}{30} \cdot T_f(t) \cdot z \cdot \frac{a_2 - a_1}{L} \quad (17)$$

where,  $n$  = rotation speed (rpm)  
 $T_f(t)$  = friction torque (Nm)  
 $z$  = motion ratio

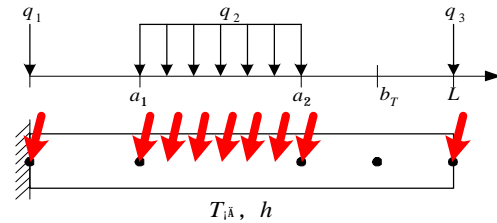


Fig. 4 Simulation model of ball screw

The operation conditions and parameters for simulation are acquired with reference to experiments by Otsuka et. al [6]. In this paper, convection coefficient is varying in time. For state space equation, 5 modes are selected when  $b$  is 0.999. Fig. 5 and Fig. 6 show the estimated heat input  $q_{nut}$  and temperatures without adaptive algorithm. Heat input is well estimated when convective coefficient is equal to the initial coefficient but not estimated well when thermal parameter of real plant is different from the one of the reference plant. Nonetheless, temperatures

are fairly estimated without adaptation algorithm except that the estimated temperature at  $x=0.25$  is not matched to the measurement when convective coefficient of real plant is different from the one of the reference plant.

Fig. 7 and Fig. 8 show the estimated heat input and temperatures by adaptive algorithm. Heat input is well estimated though convective coefficient is different from the one of reference plant. Temperatures are well estimated by adaptation algorithm at  $x=0.25$  and  $x=0.43$  even when convective coefficient of real plant is not equal to the one of the reference plant.

## 5. Conclusion

In this paper, 1-dimensional heat transfer problems were modeled with the continuous state space equation through mode reduction technique. After deriving IHCP solution in a frequency-domain, the observer was designed with the Butterworth low pass filter. And then, adaptation algorithm was applied so as to estimate the time-variant parameter in actual thermal process. The temperature distribution of a ball screw was estimated with temperature measurements in case of the presence of noise. Although the heat input was not estimated well, the temperatures are estimated fairly with only fixed observer. When adaptation algorithm is applied to the observer, the heat input and the temperatures are estimated precisely regardless of time-variant thermal parameters.

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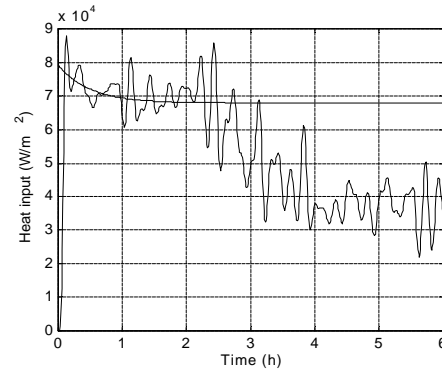


Fig. 5 Estimation of heat input by fixed observer

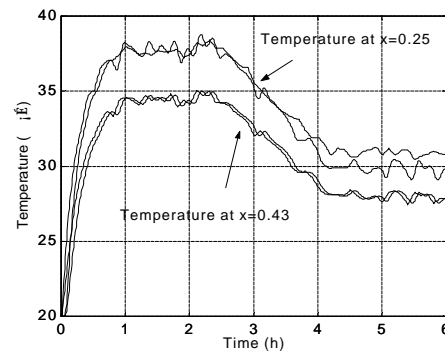


Fig. 6 Estimation of temperature by fixed observer

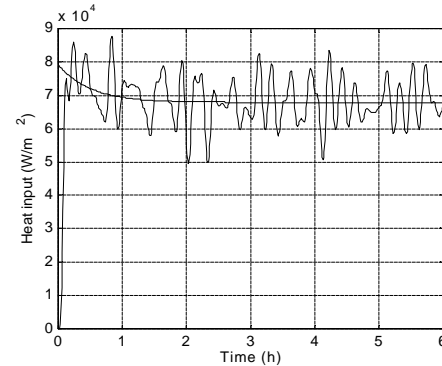


Fig. 7 Estimation of heat input with adaptation algorithm

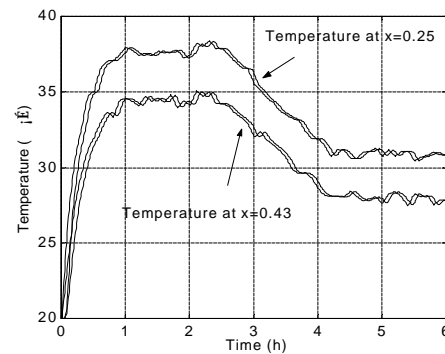


Fig. 8 Estimation of temperature with adaptation algorithm