Modeling of Geometric Errors of Manufactured Parts for Manufacturing Process Control

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Introduction

Imperfections in manufacturing processes (such as spindle errors of machine tools, thermal expansion of workpiece, and tool wear) cause machined workpieces to deviate from their ideal designed geometry. The manufactured geometric errors are usually measured with coordinate metrology devices in terms of standard geometric errors such as straightness errors, roundness errors, etc. Although these errors may have some links to product performance, they often do not have direct correlation with the imperfections (or errors) of manufacturing processes and thus they are not suitable for manufacturing process control. In the last decades, attempts have been made to manipulate or decompose the measured geometric errors into various frequency components: a high frequency component as *roughness error*, an intermediate frequency component as *waviness errors*, and a low frequency component as *form errors*. Although the manipulated error information is helpful in manufacturing process control, its effectiveness for identifying error sources of a manufacturing process has been unsatisfactory.

In this research, a new method based on *Legendre and Fourier* (*L-F*) *polynomials* is introduced for modeling geometric errors of machined cylindrical parts. The method is demonstrated through the modeling of manufactured errors in precision cylindrical turning and grinding processes. These errors and their sources are studied in three independent categories, cross-section form errors, axial form errors, and cross-section size errors. In each category, modeling of typical manufactured errors is discussed. The intention of this research is to use the novel method to better understand the root causes of the errors and provide an effective diagnosis tool for manufacturing process control.

Cross-Section Form Errors

Cross-section error of a manufactured cylindrical part is the error out of roundness in a cross section perpendicular to the axis of a manufactured part. Modeling of the errors produced by spindle defects of a machine tool and fixturing distortion is discussed.

The spindle of a lathe or a chucking-type spindle grinder is supported by a series of bearings in headstock of a machine tool. Some imperfections on bearing components make the spindle unstable in the radial direction as the spindle rotates. The radial variation of spindle causes geometric errors on the manufactured workpiece. Typical imperfections on bearing components are form errors and size errors of a bearing component such as a bearing ball, roller, or groove. The imperfections often produce sine and cosine shape errors on a machined workpiece. In an example of a ball bearing with the oversized ball by ΔD in a lathe spindle, it is not difficult to mathematically derive a geometric error function, $\Delta r(\theta)$, on the workpiece as shown in Equation (1).

$$\Delta r(\theta) \approx -\Delta D \cos\left[\frac{1}{2}\frac{R_1}{R_1 + R_2}\theta\right] \qquad (1)$$

$\Delta r(\theta)$: radial error;
ΔD: diameter error of the oversized ball;
R_1: radius of the inner groove of the bearing;
R_2: nominal radius of the oversized ball;

As in the above example, many other defects on a ball or inner groove of a journal bearing can lead to sine and cosine form errors on a machined cylindrical workpiece. If there is a dominant defect in the bearing components, a machined surface error may appear in an obvious sine or cosine pattern, such as a two-lobing shape, three-lobing shape, and so forth. In general, the geometric error function, $\Delta r(\theta)$, by the spindle bearing components on the machined surface is a sum of sine and cosine functions as Equation 2.

$$\begin{aligned} r(\theta) &= r_0 + \Delta r(\theta) \\ &= r_0 + \sum A_i \cos(i\theta) + \sum B_i \sin(i\theta) \end{aligned} \qquad (2)$$

θ: angle in cross-section;
$r(\theta)$: radius of a workpiece at θ;
r_0: average radius of a workpiece;
$\Delta r(\theta)$: radius error of a workpiece at θ;
A_i and B_i: constants.

For workpieces with low rigidity, fixturing force may cause a geometric error on the workpieces, and the error normally appears as a sine or cosine function. In turning and grinding a diameter of sleeve, for example, the geometric error caused by the force of a three-jaw chuck on the workpiece can be modeled as Equation 3.

$$\begin{aligned} r(\theta) &= r_0 + \Delta r(\theta) \\ &= r_0 + A\cos(3\theta) \end{aligned} \qquad (3)$$

Axial Form Errors

Axial error of a manufactured cylindrical part is the error out of cylindricity along a section plan passing the axis of a manufactured part. Modeling of some typical errors in cylindrical turning and grinding processes are discussed here, including errors by fixturing misalignment, workpiece deflection, and heat expansion.

When work centers on a lathe or grinder are not aligned to the feeding direction of the cutting tool or traverse direction of the grinding wheel, the misalignment will lead to cylindricity errors along the axis. If the misalignment between the axes of two work centers is offset by Δr in a horizontal plane, a taper error of Δr will be produced. It is approximately the difference of the maximum and minimum radii (see Figure 3). The taper error by the misalignment is modeled in Equation 4.

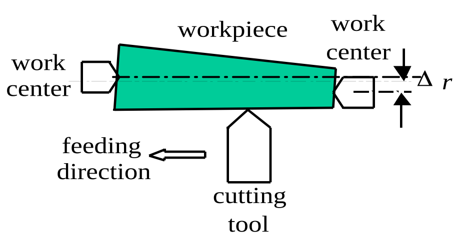

Figure 3: Taper Error by the Horizontal Misalignment

$$\begin{aligned} r(z) &= r_0 + \Delta r(z) \\ &= r_0 + Az \end{aligned} \qquad (4)$$

z: axial coordinate of the workpiece;
r_0: average radius;
A: constant.

When a shaft is turned or ground in a chuck or between work centers, cutting force causes deflection of the workpiece (see Figures 4 and 5). The deflection makes the workpiece away from the cutting tool. The degree of the deflection at cutting location (point) varies and depends on how far the cutting tool is from supporters of the chuck or work centers. The deflection at cutting location is a function of the distance of the tool from the supporters. The error function can be determined by the deflection function from the materials mechanics [3].

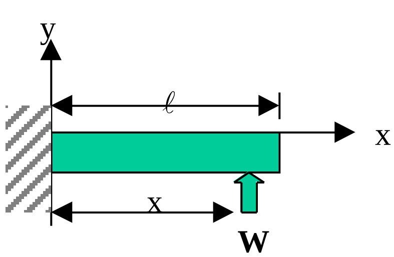

Figure 4: Workpiece in a Chuck as Beam Fixed at One End

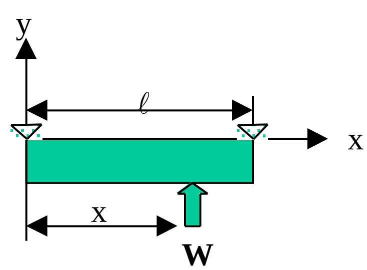

Figure 5: Workpiece Between Work Centers as Beam Supported by Both Ends

When a shaft is held in a chuck, it is viewed as a beam fixed at one end and under an intermediate load (see Figure 4). The deflection at cutting position, y, is calculated by Equation 5. The form error caused by the deflection can be rewritten in a form of polynomial functions as Equation 6. If the form error is modeled in Legendre functions, Equation 6 can be approximately replaced by Equation 7.

$$y = \frac{Wx^3}{6EI} \quad (5)\,[3]$$

E: modulus of elasticity of the material;
I: moment of inertial of cross-section of beam;
W: load on beam;
y: deflection at cutting location.

$$\begin{aligned} r(z) &= r_0 + \Delta r(z) \\ &= r_0 + Az^3 \end{aligned} \quad (6)$$

z: coordinate of workpiece axis;
$r(z)$: radius of a workpiece at z;
r_0: average radius of a workpiece;
Δr: radius error of a workpiece at z.

$$\begin{aligned} r(z) &= r_0 + \Delta r(z) \\ &= r_0 + Bz + C(3z^2 - 1)/2 \end{aligned} \quad (7)$$

In the case of a shaft hold between work centers, it is a problem where a beam is supported at both ends under an intermediate load (see Figure 5). The deflection, y, at cutting position is calculated by Equation 8. The error by the deflection in a form of polynomial functions is rewritten as Equation 9. The approximate model in Legendre functions is shown in Equation 10.

$$y = \frac{W}{3EI\ell}\left(\ell^2 x^2 - 2\ell x^3 + x^4\right) \quad (8)\,[3]$$

$$\begin{aligned} r(z) &= r_0 + \Delta r(z) \\ &= r_0 + Az^2 + Bz^3 + Cz^4 \end{aligned} \quad (9)$$

$$\begin{aligned} r(z) &= r_0 + \Delta r(z) \\ &= r_0 + D(3z^2 - 1)/2 \end{aligned} \quad (10)$$

In turning and grinding operations, energy dissipated in cutting operations is converted into heat, which, in turn, raises the temperature in the cutting zone. The temperature results in expansion in size of the workpiece. During a cutting operation, the graduate change of temperature causes a geometric form error on the workpiece. Thermal expansion in diameter of the workpiece is calculated by Equation 11. Since the temperature rising is a linear function of time [1], the form error caused by the heat will be a taper as expressed in Equation 12.

$$\Delta D = CD(T - T_0) \quad (11)\,[3]$$

ΔD: change in diameter;
D: diameter;
C: thermal conductivity;
T_0: initial temperature of the workpiece;
T: final temperature of the workpiece.

$$\begin{aligned} r(z) &= r_0 + \Delta r(z) \\ &= r_0 + Az \end{aligned} \quad (12)$$

z: coordinate of the workpiece axis;
$r(z)$: radius of the workpiece at z;
r_0: average radius of the workpiece;
$\Delta r(z)$: radius error of the workpiece at z;
A: constant.

Cross-Section Size Errors

In this study, size of a cylindrical workpiece refers to the diameter of the workpiece. Diameter of a manufactured cylinder is defined as an average diameter of the workpiece. Size (diameter or radius) error of a manufactured cylindrical part is equal to the difference between an actual size and the expected size of the manufactured part. In precision turning and grinding, many factors affect the size. The most common factors in manufacturing are the setup error and tool wear. The diameter error by the two factors can be expressed in Equation 13.

$$\begin{aligned} r(z) &= r_0 + \Delta r(z) \\ &= r_0 + A \end{aligned} \quad (13)$$

z: coordinate of the workpiece axis;
r: radius of the workpiece at z;
r_0: average radius of the workpiece;
Δr: radius error of the workpiece at z;
A: constant.

L-F Polynomials Modeling

From the above discussion, a cross-section form or size error from typical manufacturing error sources can be expressed by a Fourier function, and an axial form error can be modeled by a Legendre function. The combined geometry and size error of a manufactured cylindrical part can be represented in a form of Legendre- Fourier (L-F) polynomials as Equation 14.

$$r(\theta, z) = r_0 + \Delta r(\theta, z)$$
$$= \sum A_{0j} P_j(z) + \sum \sum [A_{ij} P_j(z) \cos(i\theta) + B_{ij} P_j(z) \sin(i\theta)] \qquad (14)$$

P: Legendre functions;
A and *B*: constants.

Conclusions

The following conclusions can be drawn from this study:

1. A geometric error of a manufactured part caused by an individual manufacturing source normally appears in a shape of one or two functions of the L-F polynomials.
2. The combined geometric error on a manufactured part is resulted from several error sources in a manufacturing system. The combined error of a manufactured cylindrical part can be decomposed into a series of L-F functions. Each L-F function normally corresponds to an error source of a manufacturing system.
3. Dominant manufactured error components can be identified by a comparison of coefficients of L-F functions. Thus, major error sources for the manufactured errors can be identified or estimated from all the error sources in the manufacturing system. The L-F polynomials modeling of manufactured errors of cylindrical parts can provide an effective tool for diagnosing and correcting errors for manufacturing process control.
4. Since dominant components of a manufactured error can be modeled by limited lower order F-L functions, computational complexity of the modeling process is relatively low.
5. The method has a good potential to be applied in an automatic system for diagnosing error sources of a manufacturing system as shown in Figure 6.

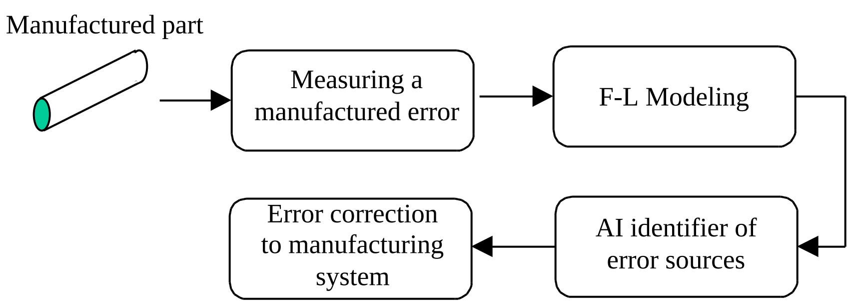

Figure 6: Error Source Diagnosis System of a Manufacturing System

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