Arbitrary N-Step Algorithm for Removal of Higher Order Test Optics Errors

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1. INTRODUCTION

Optical testing suffers some inevitable systematic instrument errors caused by the fabrication imperfection and alignment inaccuracy of constituting optical components. When the required uncertainty in the measurement is of the same order as the instrument errors, self-calibration based upon principles of absolute test becomes important since no calibration standards with sufficiently quantified accuracy are usually available. Ever since the reversal principle of three-flat test was first known, a number of efforts have been made to extend the principle to the full-aperture test of lenses and mirrors [1,2]. One approach is adopting well-defined model functions such as the Zernike polynomials or Fourier series to approximate surface errors [3-6]. The use of model functions led to the N-step averaging method, which removes the instrument errors by averaging multiple wavefronts sampled by rotating the part to N equally spaced azimuthal positions about the optical axis [7-9]. In fact the part wavefront generated by rotation may be divided into three components; the rotationally variant (RV), angular harmonic (AH) of integer multiples of N, and rotationally invariant (RI) wavefronts [9]. The N-step averaging method successfully separates the RV wavefront from the instrument errors, but this is generally not the case for the AH and RI wavefronts except for some special situations [4].

Our intention in this paper is to improve the N-step averaging method with a particular attention of effective separation of the AH wavefront. In principle, the AH wavefront can be made insignificant in the N-step averaging method by increasing the number of part rotation. In practice, however, this requires a high precision in part rotation because higher order wavefront components are particularly sensitive to rotation errors. This problem could diminish with aids of an accurate azimuthal angle division device, but a more effective way would be modifying the current computational method as done in the 'arbitrary N-step algorithm' proposed in this paper. The new algorithm is not confined by the equal spacing requirement, permitting part rotations to be made at arbitrary azimuthal positions. This generalized algorithm eliminates calibration errors caused by rotation inaccuracy and also offers a great advantage of reducing the required number of part rotations drastically when higher order spatial frequency terms are of particular importance. The latter benefit is obtained by imposing a predetermined small amount of intentional offset in the azimuthal positions during part rotations.

2. N-STEP AVERAGING METHOD

The N-step averaging method starts with the basic assumption that the measured wavefront from interferometric optical testing is a superposition of two distinguishable contributions of

$$W = T + P \qquad (1)$$

where T is the instrument error including the reference surface, while P is the part wavefront to be measured. To remove T from W, the part is rotated about the optical axis by some physical means while T remains stationary. For analysis, let W_j be the sampled wavefront of W when P is stationed at an azimuthal position of α_j with the subscript j indicating the rotation index ranging from 0 to N-1. It is assumed that $\alpha_0 = 0$ for convenience, and N is the total number of part rotations. Taking the Zernike polynomials, the part wavefront during rotation is expressed as

$$\begin{aligned} P_j(r,\theta) = P_0(r,\theta+\alpha_j) &= \sum_{l,k} R_l^k(r)[c_{0,lk}\cos(k\theta+k\alpha_j)+\tilde{c}_{0,lk}\sin(k\theta+k\alpha_j)] \\ &= \sum_{l,k} R_l^k(r)[c_{j,lk}\cos(k\theta)+\tilde{c}_{j\,lk}\sin(k\theta)] \end{aligned} \qquad (2)$$

where $c_{j,lk} \quad c_{0,lk}\cos(k\alpha_j)+\tilde{c}_{0,lk}\sin(k\alpha_j)$ and $\tilde{c}_{j,lk} \quad \tilde{c}_{0,lk}\cos(k\alpha_j)-c_{0,lk}\sin(k\alpha_j)$. Parks [3] and Fritz [4] first suggested rigorous computational procedures to determine the coefficients $c_{0,lk}$ and $\tilde{c}_{0,lk}$ directly with only two wavefront measurements. This two-step algorithm is convenient to implement but invalid in case k equals 0 or integer multiples of $2\pi/\alpha_j$. This means that only the RV part wavefront is self-calibrated, excluding the RI and AH components.

Later Evans and Kestner [7] proposed the N-step averaging algorithm, which takes equally spaced part rotations around a revolution, i.e., $\alpha_j=2\pi j/(N-1)$. In this case, the Zernike coefficients yield two useful summing properties of

$$\sum_{j=0}^{N-1} c_{j,lk} = \sum_{j=0}^{N-1} \tilde{c}_{j,lk} = 0\,, \tag{3}$$

The part wavefront consequently cancels itself out if the measured wavefronts W_j are summed up, and this leads to the final form of the N-step averaging algorithm of

$$P_o = W_o - \frac{1}{N}\sum_{j=0}^{N-1} W_j\,. \tag{4}$$

In comparison with the previous two-step algorithm, this multiple N-step algorithm provides improvement in calibration accuracy with relatively simple arithmetic averaging. However, the problem that the RI and AH part wavefronts are not separated remains unsolved since the properties of Eq.(3) are not valid if k equals zero or integer multiples of N as in the previous algorithm.

3. ARBITRARY N-STEP ALGORITHM

The loss of the AH part wavefront may be minimized if the number of part rotation N is taken large enough. In practice, however, this requires a high precision in part rotation because higher order wavefront components are particularly sensitive to rotation errors. This problem could be coped with if an accurate device of azimuthal angle division is used. Nevertheless, a more effective way would be modifying the current computational method as in the 'arbitrary N-step algorithm' proposed hereafter. The new algorithm is not confined by the equal spacing requirement to permit part rotations to be made at arbitrary azimuthal positions. The key idea is to treat the rotated angles α_j as additional unknowns together with the coefficients $c_{0,lk}$ and $\tilde{c}_{0,lk}$, and adopt the least squares technique to determine the true values of the unknowns simultaneously. For this, P_0 is rearranged in a combined form of

$$P_0(r,\theta) \equiv \sum_k P_0{}^k(r,\theta) = \sum_k \{\sum_l^{L(k)} \xi_{0,l}^k Z_l^k(r,\theta)\} \tag{5}$$

where $Z_l^k(r,\theta) \equiv R_l^k(r)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(k\theta)$ *with* $\xi_{0,l}^k$ being corresponding coefficients [11], and K is the maximum order up to which the Zernike polynomials to be identified. In addition, the wavefront difference $D_j(r,\theta)$ is newly defined as the subtraction of

$$D_j(r,\theta) = P_j(r,\theta) - P_0(r,\theta) \equiv \sum_{k=1}^{K} D_j^k(r,\theta) = \sum_{k=1}^{K}\{\sum_l^{L(k)} \Delta_{j,l}^k Z_l^k(r,\theta)\}\,. \tag{6}$$

where $\Delta_{j,l}^k \equiv \xi_{0,l}^k[\cos(k\alpha_j)-1] + \tilde{\xi}_{0,l}^k\sin(k\alpha_j)$

Now, when multiple measurements have been completed by rotation, let $\hat{D}_j^k(r,\theta)$ be the actually measured value of $D_j^k(r,\theta)$ with $\hat{\Delta}_{j,1}^k$ being the Zernike coefficients computed from $\hat{D}_j^k(r,\theta)$. Two cost functions to be minimized are then defined as

$$E_l^k = \sum_{j=0}^{N-1}\{\Delta_{j,l}^k - \hat{\Delta}_{j,l}^k\}^2 = \sum_{j=0}^{N-1}\left\{\xi_{0,l}^k[\cos(k\alpha_j)-1] + \tilde{\xi}_{0,l}^k\sin(k\alpha_j) - \hat{\Delta}_{j,l}^k\right\}^2 \quad \text{and} \tag{7}$$

$$E_j^k = \sum_l^{L(k)} \{\Delta_{j,l}^k - \hat{\Delta}_{j,l}^k\}^2 = \sum_l^{L(k)} \left\{ \xi_{0,l}^k [\cos(k\alpha_j) - 1] + \tilde{\xi}_{0,l}^k \sin(k\alpha_j) - \hat{\Delta}_{j,l}^k \right\}^2 . \qquad (8)$$

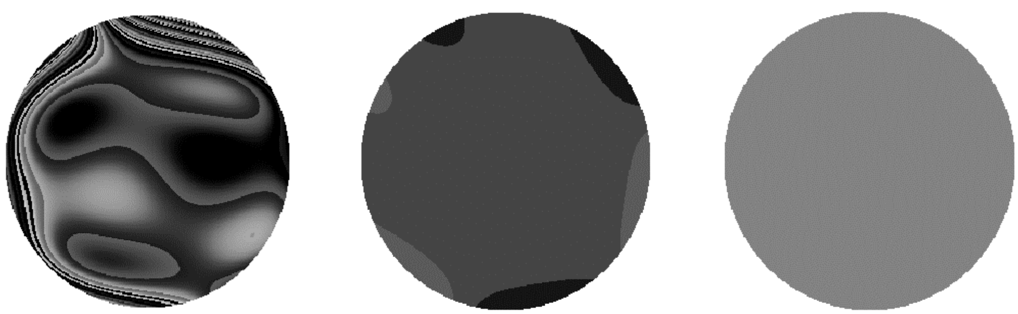
According to the rules of least squares, the necessary conditions to minimize the above cost functions are given in the form of

$$\frac{\partial E_l^k}{\partial \xi_{0,l}^k} = \frac{\partial E_l^k}{\partial \tilde{\xi}_{0,l}^k} = \frac{\partial E_j^k}{\partial \cos(k\alpha_j)} = \frac{\partial E_j^k}{\partial \sin(k\alpha_j)} = 0. \qquad (9)$$

As described in detail in [11], the partial derivatives of Eq.(8) reduce to two linear algebraic matrix equations so that the true values of $\xi_{0,1}^k$, $\tilde{\xi}_{0,1}^k$, and α_j are obtained in an iterative way until 10^{-7} order convergence is achieved.

4. SIMULATION

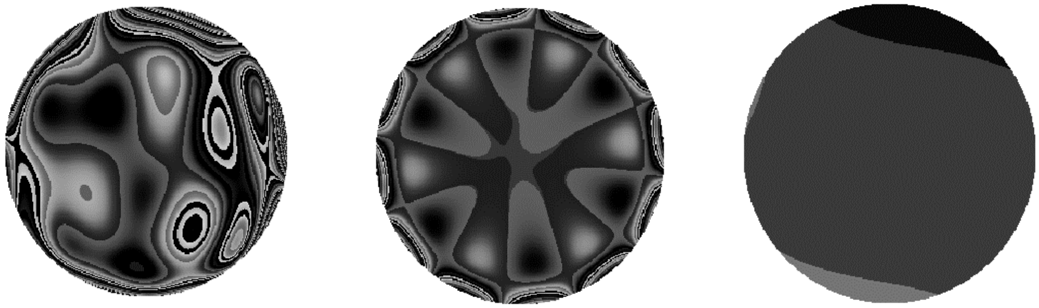
The proposed arbitrary N-step algorithm has been tested to verify its advantages and usefulness through computer simulation. Figure 1 describes a case study in which the performances of the N-step averaging and AR-step (arbitrary N-step) algorithms are compared when there are significant amounts of azimuthal position errors in part rotations. Figure 1(a) illustrates the instrument error to be eliminated while the part wavefront is assumed perfectly flat. The number of steps was taken as 6 equally for both the algorithms and the maximum Zernike order of interest was K=5. In comparison, it can be seen that the AR-step algorithm is capable of effectively removing almost all the instrument error although part rotations are not accurately induced as intended. On the other hand, the N-step algorithm is limited in restoring the part wavefront especially around the circumference of the measured area.



(a) (b) (c)

Figure 1. Comparison of simulation results when true values of α_j are not equally spaced mistakenly with 0^o, 61^o, 121^o, 179^o, 241^o, 299^o. Original wavefront generated for simulation with all the Zernike coefficients being 0.1λ for k=1-5. (a) Fringe map of the original wavefront (P-V: 1.118μm and rms: 0.110λ). (b) The wavefront error extracted by the 6-step averaging technique (P-V: 0.017μm and rms: 0.001λ). (c) The wavefront error computed by the AR-step algorithm (P-V: 0.001μm and rms value: $3 \times 10^{-7}\lambda$).

Figure 2 describes another case in which higher order instrument errors are dominant in the range of k=6 to 8 as shown in (a). If the number of part rotation is taken as 6, the N-step algorithm fails to remove the 6^{th} angular harmonic error components as illustrated in (b). On the other hand, the AR-algorithm suppress the higher order instrument errors with the same number of part rotations. Effective elimination of higher order errors are especially important when the surface irregularities of the part is to be examined accurately.



(a) (b) (c)

Figure 2 Suppression capabilities of high frequency components when α_j are intentionally taken as 0^o, 61^o, 121^o, 179^o, 241^o, 299^o. (a) Original wavefront generated for simulation with all the Zernike coefficients being 0.1λ for k=1-8 (P-V: 1.763μm and rms: 0.110λ). (b) The wavefront error extracted by the 6-step averaging technique (P-V: 0.409μm and rms: 0.001λ). (c) The wavefront error computed by the AR-step algorithm (P-V: 0.012μm and rms value: $2x10^{-5}$λ).

5. CONCLUSIONS

Our intention in this paper is to improve the N-step averaging method with a particular attention of effective separation of the AH wavefront. The 'arbitrary N-step algorithm' proposed in this paper is not confined by the equal spacing requirement, permitting part rotations to be made at arbitrary azimuthal positions. This generalized algorithm eliminates calibration errors caused by rotation inaccuracy and also offers a great advantage of reducing the required number of part rotations drastically when higher order spatial frequency terms are of particular importance. The latter benefit is obtained by imposing a predetermined small amount of intentional offset in the azimuthal positions during part rotations.

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