Sampling and Measurement Uncertainty in Coordinate Metrology

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Introduction

In order for a measurement to be complete, it is necessary to know both the value of the measurand and the uncertainty of the result. Measurement process plans generally involve a compromise between the cost of inspection and the reliability of the measurement result. Since the cost of a failed inspection is usually orders of magnitude larger than the cost of the inspection itself, the critical issue is establishing that the measurement uncertainty is sufficiently small. In the case of coordinate measurement, the sources of uncertainty can be broadly divided into the categories of machine, environment, computation and workpiece.

Uncertainties attributable to the machine are the result of errors in the point coordinates reported by the measurement system over the course of the measuring process. Methods to quantify and minimize the magnitude of these errors are well known [1-4].

Environmental uncertainty is the result of deviations in temperature, pressure, humidity and vibration which affect the measurement system. Measurement and correction for these parameters is sometimes performed but nonuniformity throughout the work volume makes the practice difficult. The most common practice is isolation of the measurement system to minimize environmental uncertainties.

Computational uncertainties are measurement errors that appear after the point data has been processed by numerical algorithms to evaluate the measurement results. They may be the result of numerical representation errors, errors in the implementation of algorithms [4], or the result of interaction between fitting software and measured deviations [6, 7].

Workpiece uncertainty is the result of the nature of the coordinate measurement process. Discrete points are collected from a continuous surface and subsequently used to represent the complete surface. If the regions between sample points contain surface defects that are not represented by the sample points, the result will be an inaccurate evaluation of the surface [8,9]. Workpiece effects are **skewed**; The reported value is always less than the true form error.

This paper presents a model for the maximum error that can result from workpiece-sampling interaction in the case of uniformly distributed samples on a 2d section. The maximum error is a function of the wavelength of the workpiece form errors, the sampling spacing and the number of sample points collected. Extension of the method to evaluate the measurement of 3d surfaces is discussed..

Problem Formulation

To quantify the influence of sampling upon measurement uncertainty, numerical tests were applied to determine the maximum measuring error that could result from uniform sampling of an arbitrary surface with systematic form deviation. Sine waves of arbitrary frequency and phase are used to simulate deterministic errors. Figure 1 shows the parameters of interest for a uniform sampling pattern on a 2D profile. The nominal shape of the surface has been removed; Thus, perfect form is represented by a line

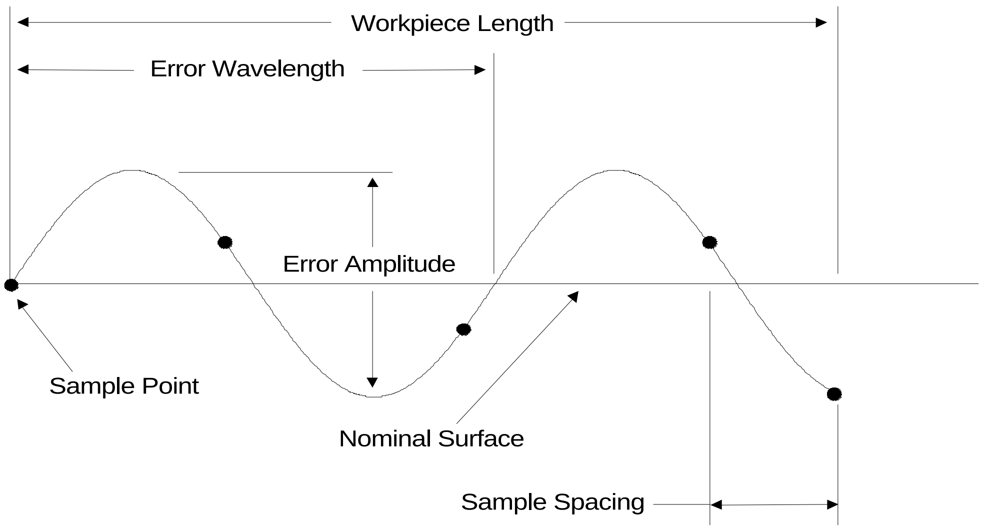

Figure 1: Sampling parameters

through the zero point. It is assumed for the purpose of this work that there is no error involved with removal of the nominal surface.

The principal parameters of the measurement are the sample spacing (φ), the number of samples (N), the error wavelength (T) and the amplitude of the error (A). Table 1 is a list of symbols used in this paper. Each measurement can be represented as a set (M) of values representing the deviation from the nominal surface. The measured deviation of the surface is the difference between the maximum and minimum values of the set M. Since the sampling pattern and the phase of the surface error are independent, the measured deviation will vary depending upon the phase relation (θ).

Table 1: List of Symbols

PARAMETER	SYMBOL
Sample Spacing	φ
Error Wavelength	T
Error Amplitude	A
Number of Samples	N
Set of Measurements	M
Phase Relation	θ
Sampling Ratio	T/θ, T_N
Measured Deviation	D
Measurement Error	ε

To simplify analysis of the problem, the parameters are non-dimensionalized by defining the amplitude of the error to be 1 and defining the ratio between the error wavelength and the sample spacing to be the sampling ratio (T_N). The sampling ratio is the number of samples per wave of the error signal and a sampling ratio of 2 corresponds to the Nyquist interval. The measurement error due to workpiece form can then be found from:

$$M = \{\theta, \theta+\varphi, .., \theta+N\varphi\} \quad (1)$$

$$D = \text{Max}[M] - \text{Min}[M] \quad (2)$$

$$\varepsilon = 1 - D \quad (3)$$

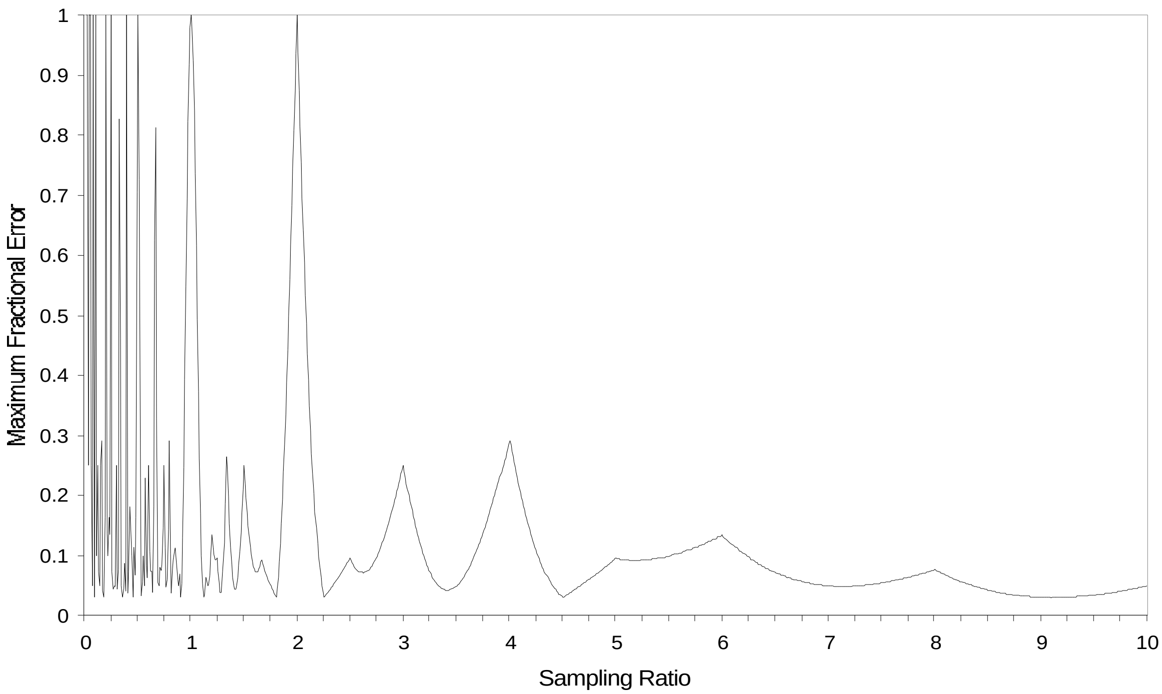

Figure 2: Maximum fraction error due to workpiece form error as a function of sampling ratio for 10 sample points.

The maximum error due to workpiece form can be found by numerically searching for the value of θ which results in the largest value of ε for a given sampling ratio and number of sample points.

Results

Figure 2 shows the influence of sampling ratio upon the maximum measuring error. The vertical axis indicates the maximum sampling error expressed as the fraction of signal amplitude that is unmeasured. The figure clearly shows that the amount of measurement error due to workpiece form decreases as the number of samples per wave grows beyond the Nyquist interval. Error wavelengths of greater than 8 times the sample spacing would be assured of being identified with less than 5% error due to workpiece form. The figure shows clear patterns between successive odd, even and fractional sampling ratios. Each error peak has a finite width; The error drops rapidly at values other than the integer ratios. The rate that the maximum error decreases as the sampling ratio changes is a function of sample size.

From the results of the numerical studies we can describe the following characteristics of maximum amplitude detection error due to workpiece form:

1. Sampling patterns that maintain a constant phase relationship with the error signal will have a nonzero maximum measurement error, regardless of sample size; Sampling ratios that can be expressed as integer fractions will have a maximum error peak.
2. The magnitude of the error peaks is dependent upon the odd or even number of samples per wave or set of waves and the sampling ratio.
3. As the sampling ratio changes from a peak value, the maximum error decreases at a rate proportional to the number of sample points and the sampling ratio.

The figure can be applied to specific measurement cases (with suitable sample sizes) by multiplying the sampling ratio by the spacing between sample points. The sampling ratio axis is then the form error wavelength in the same units as the sample spacing. Reading the maximum fractional error axis indicates the proportion of amplitude at the given wavelength that could be overlooked by the measurement process.

This method can be extended to 3d surface measurements trivially by the assumption that the surface errors can be represented by 2 sets of Fourier series in orthogonal basis directions. Such an assumption is most appropriate for parts manufactured with Cartesian coordinate systems. Surfaces that contain errors better represented by different basis functions such as suggested in the case of cylindrical holes by Henke et. Al. [11] would require the evaluation of maximum measuring error for the new basis functions. However, the variation in estimated maximum measuring error due to different basis sets is likely to be less than the variation due to the uncertainty in estimating the error frequencies that will be present.

Conclusions

A model for the maximum measurement error due to workpiece form error has been developed for uniform sampling patterns. The measurement error is a function of the ratio between the sampling interval and the form error wavelength, and the total number of samples collected

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