

# Geometric error analysis and calibration of a Stewart platform

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## INTRDOCUTION

A conventional Cartesian series-linking machine tool is known to have 21 geometric errors. Because its series-linking and stacked mechanism produce large Abbe offset length, angular errors of machine axes are considered to be the largest contributors to the volumetric errors of conventional machine tools. Another feature of a conventional machine tool is that the errors of the absolute link lengths and joint coordinates of each axis have not effect on the positioning accuracy of a machine. For a Stewart platform type machine, its parallel-linking mechanism not only reduces the Abbe error to minimum but also owns the advantage of averaging effect. The straightness and angular errors of each link has little effect on the positioning accuracy. However, a new class of errors from the universal joints or ball joints is introduced. Kinemater parameters including absolute link lengths and joint coordinates are absolutely required to determine the machine positioning accuracy. Our goal is to develop a calibration method for determining the link lengths and joint coordinates of a Stewart platform.

Because the series-linking machine is an open loop mechanism, the geometric errors of a series-linking machine must be detemined by external calibration instrument such as laser interferometers, artifacts or ball-bars. For a parallel-linking mechanism, the geometric errors can be calibrated by implicit method or self-calibration[1, 2] where additional motion information of the machine joints can be measured using redundant sensors. In this paper, we will present a self-calibration technique in which the rotations of some universal joints of the Stewart platform are measured using laser rotary encoders. Self-calibration principle and parameter estimation algorithm will be discussed.

## PRINCIPLE OF SELF-CALIBRATION

The principle of self-calibration of a closed-loop four-bar linkage described by reference [2] is depicted in Fig. 1.  $b$ , and  $c$  represent the length of links AC and AB which are fixed length.  $a$  is the initial length of link BC and  $a_k$  is displacement of the actuator in link BC.

$\alpha$  and  $\alpha_k$  are the initial value and incremental value of angle  $\angle CAB$ .  $\beta_k$  is the value of the angle  $\angle CBA$ . If  $\alpha_k$  can measured using redundant sensor and  $a_k$  is the known control-input, we are able to solve the kinemate parameters of  $b, c, a, \alpha, \beta_k$  through the kinematic constraints and the following two equations:

$$c - b \cos(\alpha_k + \alpha) - (a_k + a) \cos \beta_k = 0 \quad (1)$$

$$b \sin(\alpha_k + \alpha) - (a_k + a) \sin \beta_k = 0 \quad (2)$$

In theory, all the kinematic parameters can be determined by giving four control-inputs, ie.  $k=1\sim 4$ . Thus, we need to solve eight equations.

## SELF-CALIBRATION ON A STEWART PLATFORM

The principle of self-calibration on a Stewart platform is depicted in Fig. 2. We separate the mechanism of a Stewart platform into six closed-loops. As shown in Fig. 2, each closed-loop involves a joint at the base, a joint at the platform, and a link. We name this loop as the  $i$ th loop. Using the inverse kinamtic analysis and the self-calibration principle, we will be able to derive the following equations of this loop:

$$\cos\gamma_i [x_k \quad r_{11k} P_{ix} \quad r_{12k} P_{iy} \quad r_{13k} P_{iz} \quad -B_{ix}] + \sin\gamma_i [y_k \quad r_{21k} P_{ix} \quad r_{22k} P_{iy} \quad r_{23k} P_{iz} \quad -B_{iy}] \quad (3)$$

$$-(\bar{l}_{ik} + l_i) \sin \beta_{ik} = 0$$

$$\sin\gamma_i [x_k \quad r_{11k} P_{ix} \quad r_{12k} P_{iy} \quad r_{13k} P_{iz} \quad -B_{ix}] + \cos\gamma_i [y_k \quad r_{21k} P_{ix} \quad r_{22k} P_{iy} \quad r_{23k} P_{iz} \quad -B_{iy}] \quad (4)$$

$$+(\bar{l}_{ik} + l_i) \sin(\bar{\alpha}_{ik} + \alpha_i) \cos \beta_{ik} = 0$$

$$z_k + r_{31k} P_{ix} + r_{32k} P_{iy} + r_{33k} P_{iz} - B_{iz} - (\bar{l}_{ik} + l_i) \cos(\bar{\alpha}_{ik} + \alpha_i) \cos \beta_{ik} = 0 \quad (5)$$

where

$\alpha_i$ : the initial value of the universal joint at the base as shown in Fig. 3;

$\bar{\alpha}_i$ : the incremental value of the universal joint;

$l_i$ : the initial length of the  $i$ th link;

$\bar{l}_i$ : the incremental value of the  $i$ th link.

In theory, all the kinematic parameters can be determined by giving four control inputs, ie.  $k=1\sim 4$ . Thus, we need to solve twelve equations. The solution of the twelve equations can be obtained using Levenberg-Margquardt numerical method.

#### EXPERIMENTAL VERIFICATION

A Stewart platform (shown in Fig. 4) was built for experimental verification. CANON's laser rotary encoders were designed at the universal joint on base (shown in Fig. 5). The laser rotary encoder has a resolution of 1.28 millions of pulse/revolution. Table 1 shows the calibrated kinematic parameters of the loop 1. Table 2 shows the angular errors of the universal joints before and after calibration measured by the CANON laser rotary encoders. The result shows some accuracy improvement after calibration. However, the improvement was not good as expected. Noise of the sensor and backlash on the mechanism are considered to be the major reason. Table 3 shows the backlash values measured on the universal joints. Therefore, the backlash must be improved if better calibration accuracy is demanded.

#### CONCLUSIONS

In this work, self-calibration method of a Stewart platform using redundant sensors has been developed. From the computer simulation and experimental result, the feasibility of this approach has been proved. We found the backlash of universal joints and ball joints will be the major problem of our testing platform. This problem can be solved by adding proper preloads on the joints. However, backlash and sensor noise cannot be totally eliminated. To obtain high accuracy Stewart platform, kinematic parameter estimation algorithm must be improved. In this preliminary work, the kinematic parameters of each loop are directly solved from twelve equations obtained from four calibration positions. In the future work, kinematic parameter estimation algorithm based on least square method will be developed.

#### ACKNOWLEDGMENT

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#### REFERENCES

- [1] Charles W. Wampler, An implicit loop method for kinematic calibration and its application to closed-chain mechanisms, IEEE, 1995.
- [2] Hanqi Zhuang, Self-calibration of parallel mechanisms with a case study on Stewart platforms, IEEE, 1997.

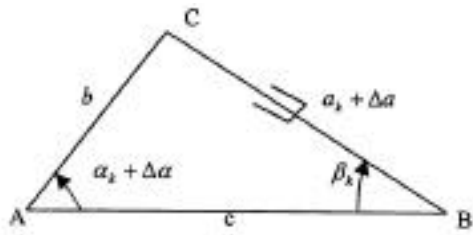


Fig. 1 : Principle of self-calibration

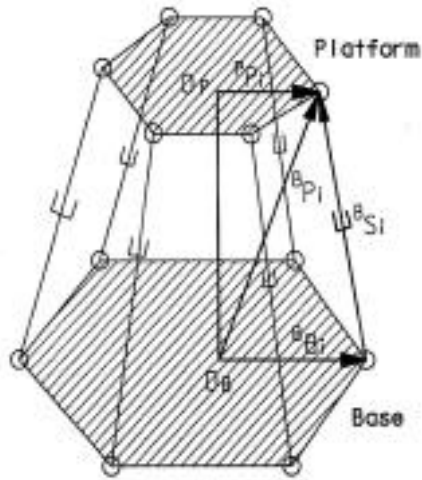


Fig. 2: Principle of self-calibration on a Stewart Platform

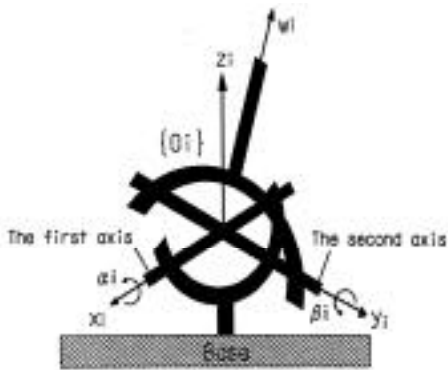


Fig. 3: The coordinate system of a universal joint at the base

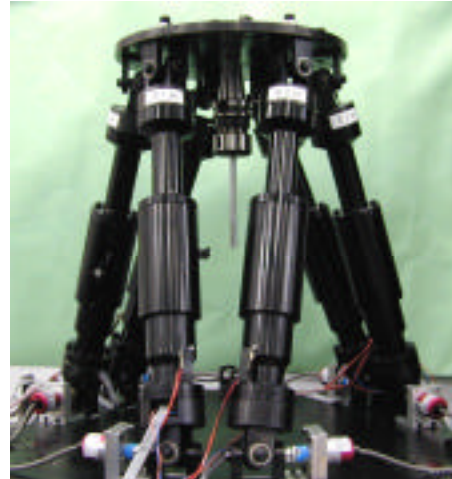


Fig.4: A Stewart platform for experimental verification

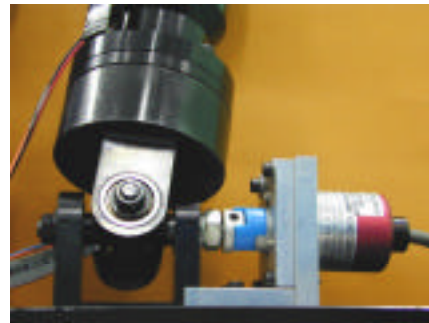


Fig. 5: The implementation of redundant sensor.

		Nominal value (mm)	Calibration (mm)
1	$B_{ix}$	50.3580	50.4264
	$B_{iy}$	285.5952	285.3575
	$B_{iz}$	38.0000	38.8386
	$P_{ix}$	89.3575	89.2791
	$P_{iy}$	106.5802	106.7169
	$P_{iz}$	-38.0000	-38.8386
	$l_i$	547.9565	546.2297

$(B_{ix}, B_{iy}, B_{iz})$ : the coordinates of the joint on the base

$(P_{ix}, P_{iy}, P_{iz})$ : the coordinates of the joint on the platform

$l_i$ : the initial length of the  $i$ th link

Table 1: Kinematic parameters of loop 1

	Before calibration (degree)	After calibration (degree)
Loop 1	0.0771	0.0817
Loop 2	0.0974	0.0790
Loop 3	0.0736	0.0254
Loop 4	0.0760	0.0264
Loop 5	0.0886	0.0715
Loop 6	0.1448	0.0273

Table 2: Mean angular errors of the universal joints on base

Loop no.	1	2	3	4	5	6
Backlash (degree)	0.084	0.059	0.005	0.006	0.040	0.059

Table 3: The backlash of the universal joint of each loop