

# Direct Measurement of Spindle Error using a double 2-point Method

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This paper presents a new double 2-point method for an in-situ spindle error measurement. It overcomes the intrinsic difficulty of the typical 3-point method, i.e., the spindle error can be obtained only after the work-piece roundness error is determined; and the spindle error measurement precision is not so high. A standard bar was tested with the new developed system. The test result is compared with that measured with the Taylor Hobson Roundness Gauge's precise rotating table and both the results agreed well with each other.

**Key words:** Spindle error, roundness, eccentric error, displacement probes, Fourier series.

## 1. Introduction

Spindle error measurement and evaluation are critical for cylindrical work-pieces. The roundness measurement can be significantly influenced by the spindle error and eccentric error especially with the case of in-process roundness measurement. The rotating accuracy of a machine tool spindle directly affects the roundness of machined parts. Numerous papers<sup>[1,2,3,4,5]</sup> dealing with this problem were published. Commonly, the spindle axis error measurement consists of: a precision arbor (a precision sphere or cylinder), reversal method, Donaldson method and typical three-point method. The accuracy of the standard arbor is poor, as the measuring signal includes both the rotational error and the shape error, which are difficult to separate. For the reversal method, it is difficult to reverse the rotation axle by 180° precisely, so the accuracy is still problematic. As to the typical three-point method, it can precisely measure the workpiece roundness without the influence of the spindle axis error, but it cannot measure the spindle axis error accurately. In this paper, a new method, called the double 2-point method, is proposed which can precisely determine the spindle error without the influence of roundness error and eccentric error.

## 2. The Principle of Radial Rotation Error Measurement

The proposed double two-point method is actually an extension of the typical 3-point method. The three sensors are still used and located at points A, B and C which are aligned with angles of 0°, 120°, and 240°, as shown in Fig.1. The measurement setup should satisfy:

- 1) Small error principle: The form error of precisely machined workpiece and spindle axis error of the rotating device are small enough compared with the size of the workpiece;
- 2) Periodicity and repetitiveness of the spindle axis error motion and constant rotating speed;
- 3) Mounting condition: Three sensors are fixed at the same circumference of the axis to be measured, in order to ensure that the workpiece shape error at the assigned point could be measured by the three probes in turn.

S1, S2, S3 in Fig.1 indicate the three displacement probes.  $O$  is the intersection of the three probes' measuring direction.  $O_0, O_1, O_2, \dots, O_n$  are the points on the spindle axis error motion trace. Supposing that the axis rotates in a clockwise direction, at the position of trigger 1,  $O_1$  is the circle center,  $A_1$  is a certain point in the measured circumference. After rotating through angle  $\theta$ , the circle center moves to  $O_2$ , and the  $A_1$  moves to  $A_2$ . If  $R(t)$  represents the roundness of the workpiece, and  $r(t)$ , the spindle error, under the above conditions, and the roundness of the workpiece should have the same and stable period for each probe, it can therefore be shown as:

$$R(t) = R(t + T), \quad T = 2\pi/\omega \quad (1)$$

The Fourier series of periodic function  $R(t)$  can be expressed as:

$$R(\theta) = R_0 + \sum_{k=1}^{\infty} R_k \sin(k\theta + \varphi_k) \quad \theta = \omega t \quad (2)$$

where amplitude  $R_k$  and phase  $\varphi_k$  are constants. The first harmonic ( $k=1$ ) is caused by the mounting error, and the high harmonics  $k \geq 2$  represents the roundness. The roundness of the case  $k \geq 2$  is the intrinsic characteristics of workpiece itself. It depends only on the machining accuracy and is independent of the used measurement method, the distribution and the number of the probes.

On the condition that the rotational angle speed is constant and the three probes are mounted around the same circumference, all roundness measured by the three probes have only some constant phase difference which are determined by the distribution of the probes. With the double two-point method, this shape error of the workpiece and the eccentric error from the measured signal can be eliminated and the spindle error can be got.

Supposing that the motion of the spindle axis error  $r(\theta)$  is quasi-periodic, it can be expanded as the following Fourier series:

$$r(\theta) = r_0 + \sum_k r_k \sin(k\theta + \rho_k) \quad (3)$$

Where  $X(\theta), Y(\theta)$  are components of  $r(\theta)$  in the direction of X and Y axes. The principle of the typical three-point method can be described by:

$$\begin{aligned} S_1(\theta) &= R_1 - R(\theta) - Y(\theta) \\ S_2(\theta) &= R_2 - R(\theta - \phi) - Y(\theta)\cos\phi + X(\theta)\sin\phi \\ S_3(\theta) &= R_3 - R(\theta - \tau) - Y(\theta)\cos\tau + X(\theta)\sin\tau \end{aligned} \quad (4)$$

Where  $R_1, R_2, R_3$  are defined as the initial radial distances of each probe at the position while the roundness  $R(\theta)$  and error motion  $X(\theta), Y(\theta)$  equal to zero;  $\phi$  and  $\tau$  represent the angles of probe  $S_1$  to  $S_2$  and probe  $S_1$  to  $S_3$  respectively. After choosing the proper coefficients  $a$  and  $b$  to satisfy

$$\begin{aligned} a \sin\phi - b \sin\tau &= 0 \\ 1 + a \cos\phi + b \cos\tau &= 0 \end{aligned} \quad (5)$$

The resultant signal  $S(\theta)$  is obtained:

$$\begin{aligned} S(\theta) &= S_1(\theta) + aS_2(\theta) + bS_3(\theta) \\ &= (R_1 + aR_2 + bR_3) - R_0(1 + a + b) \\ &\quad + X(\theta)(a \sin\phi - b \sin\tau) - Y(\theta)(1 + a \cos\phi + b \cos\tau) \\ &\quad + \sum_{k=1} \left\{ [A_k(1 + a \cos k\phi + b \cos k\tau) + B_k(b \sin k\tau - a \sin k\phi)] \cos k\theta \right. \\ &\quad \left. + [B_k(1 + a \cos k\phi + b \cos k\tau) - A_k(b \sin k\tau - a \sin k\phi)] \sin k\theta \right\} \\ &= (R_1 + aR_2 + bR_3) - R_0(1 + a + b) \\ &\quad + \sum_{k=2} [(A_k \alpha_k + B_k \beta_k) \cos k\theta + (B_k \alpha_k - A_k \beta_k) \sin k\theta] \end{aligned} \quad (6)$$

where:

$$\begin{aligned} \alpha_k &= 1 + a \cos k\phi + b \cos k\tau \\ \beta_k &= b \sin k\tau - a \sin k\phi \end{aligned} \quad (7)$$

By expanding  $S(\theta)$  into the Fourier series, we get:

$$S(\theta) = S_0 + \sum_k (F_k \cos k\theta + G_k \sin k\theta) \quad (8)$$

$$\text{and } \begin{aligned} A_k &= \frac{F_k \alpha_k - G_k \beta_k}{\alpha_k^2 + \beta_k^2} \\ B_k &= \frac{F_k \beta_k + G_k \alpha_k}{\alpha_k^2 + \beta_k^2} \end{aligned} \quad (9)$$

the result of equation (9) shows that:

- 1) If,  $\alpha_k^2 + \beta_k^2 \neq 0$  roundness can be measured precisely without the effect of the spindle error and the mounting error of the workpiece;
- 2) Roundness does not include the eccentric error of the workpiece in case  $k=1$ , so that the typical three-point method can not remove this eccentric error caused by the workpiece mounting;

So, from equation (5):

$$\begin{aligned}\bar{X}(\theta) &= \left\{ [S_2(\theta) - \bar{R}(\theta - \phi)] \cos \tau - [S_3(\theta) - \bar{R}(\theta - \tau)] \cos \phi \right\} / \sin(\phi - \tau) \\ \bar{Y}(\theta) &= \bar{R}(\theta) - S_1(\theta)\end{aligned}\quad (10)$$

The first harmonic representing eccentric error is incorrectly added to  $\bar{X}(\theta)$ ,  $\bar{Y}(\theta)$ , and it damages the precision of the measured spindle axis error, especially for checking high accurate spindle.

The double 2-point method makes use of the phase differential of the three probes  $S_1, S_2, S_3$ , e.g.,  $S_1-S_2$  and  $S_1-S_3$ . By doing the differential, the spindle axis error can be obtained directly with the removal of the effect of workpiece roundness as well as eccentric error. The principle can be described with the following equations:

$$\begin{aligned}S_1(\theta) &= [R(\theta)]_1 + [r(\theta)]_1 \\ &= R_1 - [R(\theta + \beta) + r(\theta + \beta) \cos(\theta + \beta)] \\ &= [R_1 - R_0 - r_0 \cos(\theta + \beta)] - \sum_{k=1} R_k \sin[k(\theta + \beta) + \psi_k] \\ &\quad - (r_k/2) \left\{ \sin[(k+1)(\theta + \beta) + \varphi_k] - \sin[(k-1)(\theta + \beta) + \varphi_k] \right\}\end{aligned}\quad (11)$$

$$\begin{aligned}S_2(\theta - \phi) &= [R(\theta)]_2 + [r(\theta)]_2 \\ &= [R_2 - R_0 - r_0 \cos(\theta + \beta)] - \sum_{k=1} R_k \sin[k(\theta + \beta) + \psi_k]\end{aligned}\quad (12)$$

$$\begin{aligned}S_2(\theta - \phi) - S_1(\theta) &= (R_2 - R_1) \\ &\quad + (r_k/2) \left\{ \sin[(k+1)(\theta + \beta) + \varphi_k] - \sin[(k-1)(\theta + \beta) + \varphi_k] \right\} \\ &\quad - (r_k/2) \left\{ \sin[(k+1)(\theta + \beta) + \varphi_k - k\phi] - \sin[(k-1)(\theta + \beta) + \varphi_k - k\phi] \right\}\end{aligned}\quad (13)$$

So, the difference between the two signals of probes  $S_1$  and  $S_2$  is the function of the phase angle  $\phi$ , and the angle can be precisely determined previously. With this result, the effects of the roundness and the eccentric error of the workpiece can be eliminated, and the spindle axis error can be directly obtained by using following expression:

$$\begin{aligned}S_2(\theta - \phi) - S_1(\theta) &= (R_2 - R_1) \\ &\quad + \sum_{n=1} A_n (n/2) \left\{ \sin[(n+1)(\theta + \beta) + P_n + \varphi_n] + \sin[(n-1)(\theta + \beta) + P_n + \varphi_n] \right\} \\ A_n &= \sqrt{2[1 - \cos(n\phi/2)]} \\ P_n &= \text{Atg} \left\{ \sin(n\phi/2) / [1 - \cos(n\phi/2)] \right\}\end{aligned}\quad (14)$$

If angle  $\phi$  is chosen properly, the calculation of  $[S(\theta - \phi)]_2 - [S(\theta)]_1$  will be easier. Similarly,  $[S(\theta - \phi)]_3 - [S(\theta)]_1$  can be obtained. An inherent problem with the typical 3-point method is that it is "blind" to the components at certain frequencies. By using these two phase differentials  $[S(\theta - \phi)]_2 - [S(\theta)]_1$  and  $[S(\theta - \phi)]_3 - [S(\theta)]_1$ , this blind problem can be solved, and therefore the spindle axis error can be precisely determined.

### 3. Experimental Result

A specially designed frame was adopted to fix the three capacitance micrometer probes. The resolution of the capacitance micrometer is  $0.02\mu\text{m}$ . The experiment was carried out for a standard bar, and the test result was verified by comparing the experimental result with that obtained from the Taylor Hobson Roundness Gauge's precise rotating table. The radial error of the spindle was evaluated by the least square method. The accuracy of the Taylor Hobson Roundness Gauge is  $0.1\mu\text{m} + 0.0005\mu\text{m}/\text{mm}$ , and the result obtained with the double 2-point

method is  $r_{\max}=0.1030098$ ,  $r_{\min}=0.000460$ , radial error of the spindle =  $r_{\max} - r_{\min} = 0.102638$ . These results verify the accuracy and reliability of the double two-point method.

#### 4. Summary and Conclusion

A new method called the double 2-point method for spindle error measurement is proposed in this paper. It is able to separate the radial error of the spindle from the roundness error of the workpiece;

During the separation of the spindle error from the roundness error, eccentric error of the workpiece was accurately eliminated and high accuracy of the spindle error measurement was obtained;

Spindle error estimation has been experimentally realized, and the effectiveness of the double 2-point method was verified by the experimental results.

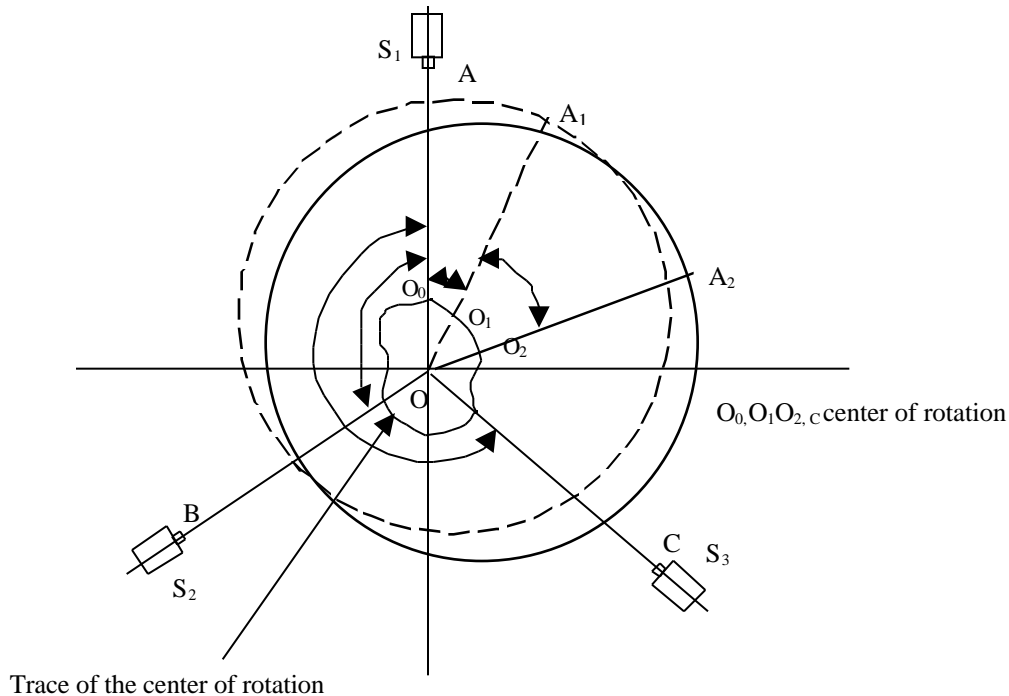


Fig. 1 The Schematic Diagram of Rotation

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