SELF-COMPENSATION OF ATMOSPHERIC-REFRACTION ERRORS IN OPTICAL METROLOGY*

Lyle G. Shirley
MIT Lincoln Laboratory, Lexington, MA

INTRODUCTION

Many measurements in precision engineering rely on the assumption that light travels in a straight line through air. Examples of these measurements include certain methods for determining errors of motion in precision machines, measuring straightness, and monitoring surface deformations of large structures. Land surveying is one application that relies heavily on this assumption. It is well-known, however, that temperature, pressure, or humidity variations acting perpendicular to the direction of propagation cause light rays to be refracted into a curved path [1]. This curvature can produce relatively large deviations between the expected and actual ray positions. For instance, a transverse temperature gradient of 10°C/m produces a beam deviation of approximately 0.5 mm after 10 m of propagation. Because the beam deviation is proportional to the square of the distance traveled, this deviation increases to 2 mm after 20 m of propagation through the same gradient. Clearly, atmospheric refraction is an important effect that must be accounted for in high-precision optical measurements.

In principle, one could monitor parameters affecting beam curvature along the beam path and calculate corrections based on these measurements [2]. Besides being cumbersome to implement and requiring access to the beam path, this approach fails to produce reliable results because of the difficulty of producing accurate closely spaced measurements of quantities such as temperature gradients along the propagation path. Another approach to compensation relies on dispersion, or wavelength dependence, in the index of refraction of air [3]. In this approach, a correction is estimated by observing the displacement between the endpoints of two rays of differing wavelength. The resulting displacement between rays, however, is only a small fraction of the total refractive error. In fact, there is less than a 2.5% variation in the magnitude of the refractive error over the visible spectrum. Consequently, this approach is prone to errors because it requires precise measurements of a second-order effect in order to calculate the correction. Although this approach has met with some success, its complexity and expense limit its applications.

The ideal measurement approach would compensate for atmospheric refraction automatically without the need for auxiliary measurements or calculations. A novel approach to atmospheric-refraction compensation is presented here that achieves this goal by automatically correcting for the dominant component of refractive error. This approach is based on optical interference and the concept of controlling the paths of interfering components such that phase errors produced by atmospheric refraction cancel. This approach represents a fundamental departure from more traditional measurement approaches such as sighting the position of a target with a theodolite or measuring deviations with respect to a laser beam.

The work reported here was motivated by the need to monitor deformations of the primary surface of the Large Millimeter Telescope (LMT) being constructed in Mexico [4]. The LMT represents a new class of radars and radio telescopes that actively maintain antenna shape against outside influences [4–6]. These influences include, for example, the variation of gravitational distortion with elevation angle, bending and vibration due to antenna motion or wind, and thermal distortions due to temperature variations of the surrounding air or radiative heating of the structure from the sun.

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*This work was sponsored by the Defense Advanced Projects Agency and the Lincoln Laboratory Advanced Concepts Program under Air Force contract F19628-95-C-0002. Opinions, Interpretations, conclusions, and recommendations are those of the author and not necessarily endorsed by the U. S. Air Force.

DISPLACEMENT SENSING

Before describing the atmospheric-compensation technique, it is necessary to summarize the underlying concept for measuring surface displacement [5,7–8]. Figure 1 depicts two mutually coherent point sources obtained by splitting a laser beam of wavelength λ. Light from the two sources, which are separated by the distance a, expands slowly and produces a sinusoidal interference-fringe pattern in the area of overlap. A corner cube is attached to the surface whose out-of-plane displacement along the z-axis is to be measured. The corner cube reflects the light back towards the sources, where it strikes a beam splitter (BS) that directs the light to a detector or detector array. (Alternately, the detector is placed on the surface being monitored.) The interference-fringe period at the detector is $d = 2\lambda R/a$, where R is the distance between the source and the corner cube. (The factor of two accounts for round-trip propagation. This factor is absent if the detector is located on the surface.) A displacement z_0 along the z-axis of the surface being monitored produces a displacement of the fringe pattern at the detector of $2z_0$. Thus, if d is known, surface displacements can be monitored by observing motion of the fringe pattern at the detector.

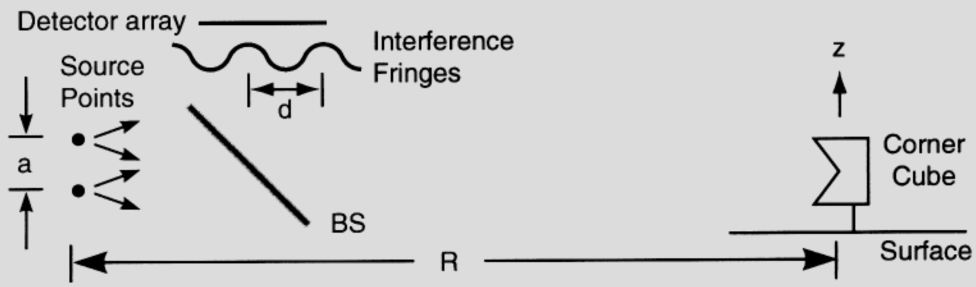

FIGURE 1. Physical basis of displacement-sensing technique.

In a system based on this concept, the fringe period d can be set such that expected surface displacements fall within one fringe period. For example, with a wavelength of $\lambda = 0.7$ µm and a point separation of $a = 2.8$ mm, the fringe period at a distance of $R = 20$ m is $d = 10$ mm. Alternately, motions exceeding the fringe period can be measured using, for example, fringe counting. It is possible in practice to determine fringe motion to better than one part in one thousand, providing micron-level displacement measurements at distances of tens of meters. The basic technique is readily extended to include many simultaneous displacement measurements distributed over a surface.

ATMOSPHERIC REFRACTION ERRORS

Although the above measurement concept has many positive features, including simplicity, robustness, speed, versatility, and minimal cost, it does suffer from the basic limitation described above—atmospheric refraction. It should be pointed out, however, that because the two beams that interfere to produce a fringe pattern travel, essentially, over a common path, the fringes are quite insensitive to other temperature-related effects. For instance, a bulk change in temperature of the air mass or temperature gradients parallel to the propagation path of the laser beam have minimal effect on the fringe pattern. As described above, the atmospheric effect that limits accuracy is the bending of light rays due to temperature gradients in the z direction—the direction in which displacement is to be measured.

Before describing the compensation technique, it is beneficial to calculate the degree of error introduced into a displacement measurement by atmospheric refraction. For simplicity, we assume there is no corner cube, and the detector is attached directly to the surface whose displacement is being monitored. In Figure 2, z_0 represents the displacement of the measurement point on the surface, and s_1 and s_2 are the optical path lengths of rays from each of the point sources to the measurement point. The objective of the analysis is to determine the motion (induced by atmospheric refraction) of the fringe pattern at z_0. Fringe motion corresponds to variations in the optical phase difference between the interfering components from the two source points. These variations in optical phase are directly proportional to the optical path length difference (OPD), given by s_2-s_1. Thus, the refractive error can be analyzed by determining the effect of temperature gradients on the OPD.

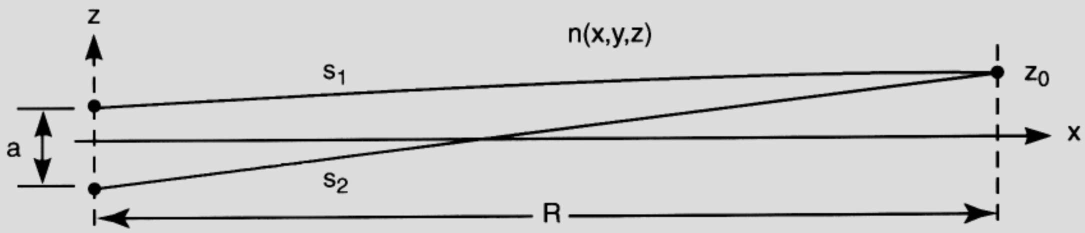

FIGURE 2. Coordinate system for calculating atmospheric-refraction errors.

To calculate the OPD, we first determine the curved ray trajectories that the two rays in Figure 2 take through the varying index of refraction n. Because deviations from the x-axis are small, the paraxial ray equations [9]

$$\frac{d^2 z(x)}{dx^2} = \frac{1}{n_0}\frac{\partial n}{\partial z} \ , \tag{1}$$

and

$$\frac{d^2 y(x)}{dx^2} = \frac{1}{n_0}\frac{\partial n}{\partial y} \ , \tag{2}$$

provide an excellent approximation to the ray trajectory. In these equations, n_0 is the average index of refraction and y points into the page in Figure 2. Once the ray paths have been determined, the optical path lengths s_1 and s_2 can be found by multiplying each length element along the ray trajectory by the value of the index of refraction at that location and integrating, yielding:

$$s = \int_0^R n\big(x, y(x), z(x)\big)\sqrt{1 + \left(\frac{dy(x)}{dx}\right)^2 + \left(\frac{dz(x)}{dx}\right)^2}\, dx \ . \tag{3}$$

The radical in Eq. (3) accounts for the slope with respect to the x-axis of the length element.

To proceed further with the calculation, it is necessary to specify the form of the index of refraction gradients $\partial n/\partial z$ and $\partial n/\partial y$ in Eqs. (1) and (2). The gradient $\partial n/\partial y$ does not produce an apparent motion of the fringe pattern because it displaces the fringes along their direction of orientation. Consequently, this gradient does not significantly affect the measurement of fringe displacement, and we set it to zero. For the purpose of illustration, we set the other gradient $\partial n/\partial z$ to a constant value P_0 that does not depend on the x position along the beam path. In a Fourier-series expansion of the x dependence of the general gradient $\partial n/\partial z$, we would refer to this uniform component as the DC component. In practice, the DC component is likely to be the leading source of refractive error because it acts along the entire path length with the same sign. By contrast, high-frequency Fourier components of the index-of-refraction gradient tend to produce a small error because these components vary sinusoidally in x and have zero mean.

Even if a uniform index-of-refraction gradient P_0 is assumed, the form of the OPD resulting from Eqs. (1)–(3) is complex and difficult to interpret. A simple and useful expression for the OPD, however, is obtained by noting that, in practice, the point-source separation a is small compared with the path length R. Thus, an accurate expression for the OPD can be obtained from the lowest-order terms of a power series expansion for small a. The resulting OPD expression for a uniform gradient P_0 is

$$\text{OPD} = \frac{an_0}{R} z_0 - \frac{P_0 R}{2} a \quad . \tag{4}$$

The solution of Eq. (4) for z_0 is

$$z_0 = \text{OPD}\frac{R}{an_0} + z_{\text{error}} \quad , \tag{5}$$

where

$$z_{\text{error}} = \frac{1}{2}\frac{P_0}{n_0}R^2 \quad . \tag{6}$$

The index of refraction gradient can be approximated in terms of the temperature gradient as [10]

$$P_0 \approx \frac{\partial T}{\partial z}\frac{\partial n}{\partial T} \approx -\frac{\partial T}{\partial z} 9.3\times10^{-7}/°\text{C} \quad \cdot \tag{7}$$

It should be pointed out that although Eq. (6) was derived for fringe motion, the same result is obtained for displacement. Thus, the numerical example given in the introductory paragraph applies equally well to fringe patterns and optical rays.

COMPENSATION

The objective of compensation is to eliminate the error term in Eq. (5). There is, however, no combination of experimental parameters that will achieve this objective and produce a measurement of z_0 (other than requiring P_0 to vanish). Obviously, a different approach to the problem is required. Suppose that instead of observing the fringe pattern associated with the interference of the rays depicted in Figure 2, we have a means of selecting the ray paths shown in Figure 3 and causing the two separated rays at the $x = R$ plane to overlap and interfere. A repetition of the above analysis of the OPD, but for the situation described by Figure 3, yields

$$\text{OPD} = \frac{(a+b)n_0}{R} z_0 - \frac{P_0 R}{2}(a-b) \quad . \tag{8}$$

Thus, the refractive error associated with this measurement is

$$z_{\text{error}} = \frac{1}{2}\frac{P_0}{n_0}\frac{a-b}{a+b}R^2 \quad . \tag{9}$$

Note that Eq. (9) reduces to Eq. (6) when $b = 0$. By inspection of Eq. (9), we see that the refractive error can be eliminated by requiring that $b = a$.

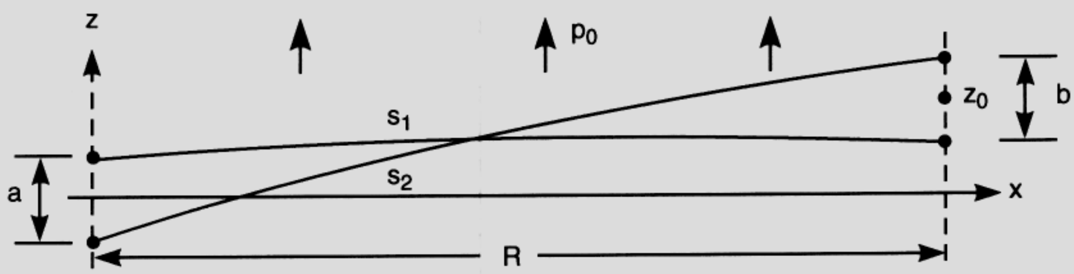

FIGURE 3. Physical basis for atmospheric-refraction compensation.

The above compensation technique has a simple intuitive explanation. Because the index-of-refraction gradient P_0 is uniform along the x-axis, the contribution to the OPD at a given value of x due to the refractive error is proportional to the vertical separation of the two rays at the point x. Thus, the rate of buildup of refractive error gradually decreases as the rays propagate from the two sources to the crossing point. At the crossing point, the positions of the rays switch, causing the sign of the contribution to the refractive error to reverse. By the time the rays have propagated to the $x = R$ plane, the cumulative error has been completely canceled due to symmetry (assuming a uniform gradient P_0).

There are many possible methods of selecting the crossed ray paths shown in Figure 3 and causing them to interfere. One class of techniques relies on polarization coding—accomplished by cross polarizing the beams emanating from the source points. The rays coming from the desired source can then be selected for each endpoint by using an appropriate pair of crossed analyzers.

In practice, it is desirable to co-locate the sources and the detector, as is done in the original non-compensated system depicted in Figure 1. Compensation can be achieved with this constraint by causing the rays to follow nearly the same path on the return trip to the detector. Thus, the rays traveling in either direction cross in the middle, and both outgoing and incoming paths are compensated.

Figure 4 depicts an optical configuration for achieving compensation with a co-located source and receiver. Here, polarization coding is achieved with a birefringent crystal, such as calcite, which separates an incident laser beam into cross-polarized ordinary and extraordinary rays. (The ordinary ray obeys Snell's law of refraction at the crystal interface; the extraordinary ray does not.) In Figure 4, an ordinary and an extraordinary ray are selected such that they cross midway between the source/receiver and the target assembly. (In general, these two rays are not generated from a single ray coming from the point source.) The target assembly consists of a polarizing beam splitter (PBS) having offset corner cubes attached to two faces. The PBS reflects the ordinary ray and transmits the extraordinary ray, or vice versa, so that each ray strikes the vertex of the corresponding corner cube. The offset of the corner cubes is set such that the distance b between the two rays entering the PBS is equal to the distance a between the two rays leaving the calcite crystal. The return rays follow the same path back through the system and are reflected by the beam splitter (BS) onto a CCD array. Because the two rays are cross polarized, it is necessary to place an analyzer before the CCD array to select a common polarization component. Otherwise, the two contributions will not interfere.

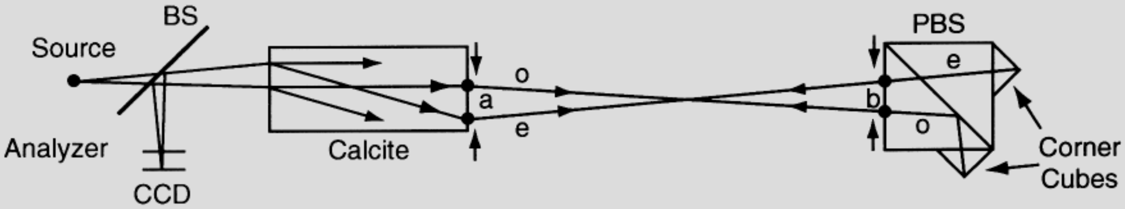

FIGURE 4. Apparatus for achieving atmospheric-refraction compensation.

The ability of the system depicted in Figure 4 to compensate for atmospheric refraction has been demonstrated experimentally [11]. This particular implementation has the advantage that the separation b can be varied by changing the offset between the corner cubes. Doing so varies the crossing point and may make it possible to optimize the system for index-of-refraction gradients whose magnitude varies along the x-axis. Research is currently under way to use the additional information obtained from additional crossing points to make higher-order refractive-error corrections.

CONCLUSIONS

A new technique has been introduced that automatically compensates for the dominant source of refractive errors in many precision optical measurements. This technique, which requires no auxiliary measurements or calculations, is simple to implement and requires no cabling or electrical communication with the target. The compensation technique can be incorporated into existing measurement techniques and provides a means for overcoming a fundamental source of measurement error that has long plagued optical metrology.

ACKNOWLEDGMENTS

This work was sponsored by the Defense Advanced Research Projects Agency and the Lincoln Laboratory Advanced Concepts Program. The author gratefully acknowledges the contributions and support of M.P. Kavalauskas, D.L. Feldkhun, L.B. Spence, J.C. Chow, and J.A. Tabaczynski.

REFERENCES

1. *Refractional influences in astronomy and geodesy*, International Astronomical Union Symposium. No. 89, E. Tengstrom and G. Teleki, eds., D. Reidel, Dordrecht, Netherlands, 1979.
2. O.A. Mozzhukhin, "Correction of leveling by refraction determined from meteorological measurements," Acta Geodaetica, Geophysica et Montanistica, **14**, 81-91 (1979).
3. L. Zeng and H. Matsumoto, "Compensation of the air turbulence by two-color method for straightness measurement," J. of the Japan. Society of Precision Engineering/Seimitsu Kogaku Kaishi **60**, 111–113 (1994).
4. G. Cortez-Medellin and P.F. Goldsmith, "Analysis of active surface reflector for a large millimeter wave radio telescope," IEEE Trans. on Antennas and Propagation, **42**, 176–183, (1994).
5. J.F. Cannon, J.A. Ball, J.E. Salah, L.G. Shirley, "A new high-efficiency surface for the Haystack antenna at millimeter-wavelengths," MIT Haystack Observatory Technical Report, February 15, 1996.
6. R.J. Lacasse, "Green Bank telescope active surface system," in *Telescope Control systems III*, H. Lewis, ed., Proc. SPIE **3351**, 310-319, (1998).
7. L.G. Shirley, "Optical sensor for active control of antenna surfaces," in Proceedings of the 1999 IEEE Radar Conference, pp. 32-35, Waltham, Massachusetts, April 20-22, 1999.
8. US Patent 5,900,936, L.G. Shirley and G.R. Hallerman, "Method and apparatus for detecting relative displacement using a light source," May 4, 1999.
9. *Handbook of Optics*, Vol. 1, p. 1.23, M. Bass, ed., McGraw-Hill, New York, (1995).
10. B. Edlen, "The refractive index of air," Metrologia **2**, 71-80 (1966).
11. L.G. Shirley and M.P. Kavalauskas, "Self-compensation of atmospheric-refraction errors: method and results," to be submitted to Appl. Opt.